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EXERCISES
IN
PRACTICAL PHYSICS

GREGORY AND SIMMONS

PART I

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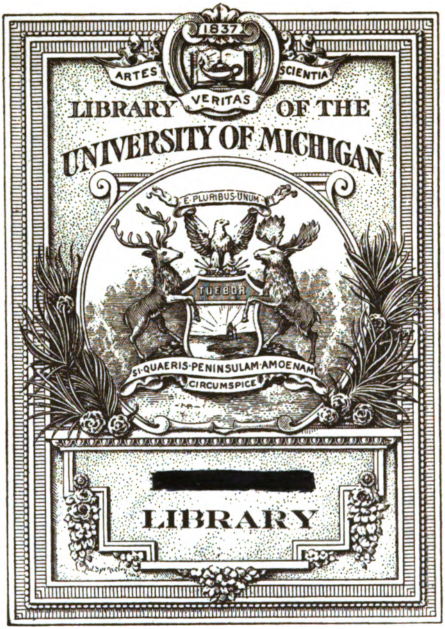

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EXERCISES IN PRACTICAL PHYSICS
FOR SCHOOLS OF SCIENCE

EXERCISES IN
PRACTICAL PHYSICS
FOR SCHOOLS OF SCIENCE

BY

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IN TWO PARTS

PART I. FIRST YEAR'S COURSE

London
MACMILLAN AND CO., LIMITED
NEW YORK: THE MACMILLAN COMPANY
1899

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GLASGOW: PRINTED AT THE UNIVERSITY PRESS
BY ROBERT MACLEHOSE AND CO.

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PREFACE.

THE characteristics of this volume and its sequel, to which we invite the attention of teachers of practical physics, are briefly:

(1) The number and variety of exercises used to exemplify each of the principles dealt with.

(2) The limitation of the text to instructions necessary for the intelligent performance of the experiments and the recognition of the significance of the results obtained.

(3) The number of new and simple experimental devices which, by the courtesy of various teachers, it has been possible to incorporate.

(4) The numerous illustrations showing at a glance the apparatus required and the method of procedure.

Teachers of practical physics, whether in "Schools of Science," or other schools in which experimental work in physical science forms a part of the curriculum, may be left to judge for themselves as to the value of these points. Our own experience in teaching and examining has convinced us that a sound knowledge of any scientific principle is only obtained after many experiments designed to present it under different aspects have been performed and studied. It is for this reason that several exercises are given under each division of the subject. When time permits a student ought to perform practically all the exercises, in order to impress upon his mind the principles they exemplify; he should, in fact, regard the exercises as he does the examples in his mathematical books, and work as many of them as he can. But where certain subjects must be studied in the very few hours per week allotted to practical physics it will be necessary for the teacher to make a selection.

The descriptive matter has been curtailed because students only require, in a laboratory manual, instructions to guide them in carrying out experimental work—they can study the theoretical aspects of their laboratory practice from lecture-notes and in text books of general physics.

We are glad of this opportunity of making very grateful acknowledgments—to the teachers whose names are mentioned during the course of the book, for new and ingenious experiments to elucidate particular principles; to Mr. L. M. Jones, B.Sc., St. Dunstan's College, Catford, for very valuable assistance while the sheets were passing through the press, and many useful hints and notes which have enabled us to improve several parts of the course; and finally, to Mr. E. Edser, Mr. H. E. Hadley, B.Sc., and Mr. James Quick for reading some of the sections and giving us the benefit of their criticisms.

R. A. G.
A. T. S.

LONDON, *August*, 1899.

CONTENTS.

CHAPTER PAGE

I. MEASUREMENT OF LENGTH, - - - - - - 1

II. MEASUREMENT OF AREA, - - - - - - - 26

III. MEASUREMENT OF VOLUME AND CAPACITY, - - 50

IV. MEASUREMENT OF MASS AND DENSITY, - - - 67

V. HYDROSTATIC METHODS OF DETERMINING DENSITIES, 72

VI. PRESSURES OF LIQUIDS AND GASES, - - - - 88

VII. PLOTTING OF CURVES, - - - - - - 104

VIII. THE LEVER, PARALLEL FORCES, AND CENTRE OF GRAVITY, - - - - - - - - 110

IX. MEASUREMENT OF TIME AND ANGLE, - - - 130

X. PARALLELOGRAM AND TRIANGLE OF FORCES, FRICTION, AND INCLINED PLANE, - - - - 136

XI. MOTION, AND THE FORCE OF GRAVITY, - - - 157

XII. THERMOMETERS, AND SIMPLE OBSERVATIONS WITH THEM, - - - - - - - - - 169

XIII. CONDUCTION AND RADIATION OF HEAT, - - - 184

ADDITIONAL EXERCISES, - - - - - - 192

INDEX, - - - - - - - - - 198

EXERCISES IN PRACTICAL PHYSICS.

CHAPTER I.

MEASUREMENT OF LENGTH.

1. Use of a Foot-rule.

(*a*) You are provided with a foot-rule. Measure a length of three feet along the edge of your table or desk. What is this length called?

Into how many *numbered* parts is the rule divided? What are these parts, or fractions, of a foot called?

> Enter your answers in a note-book in words similar to the following:
>
> A length of 3 feet is called a ....
>
> There are ... numbered parts upon my foot-rule. Each of these parts is ...th of a foot, and is called an ....

(*b*) Find, by counting divisions on your rule, how many eighths of an inch there are in the following:

		No. of eighths.
One quarter ($\frac{1}{4}$) of an inch	=	
One-half ($\frac{1}{2}$) of an inch	=	
Three-quarters ($\frac{3}{4}$) of an inch	=	

(*c*) Count how many sixteenths of an inch there are in the following:

		No. of sixteenths.
One-half ($\frac{1}{2}$) of an inch	=	
Three-quarters ($\frac{3}{4}$) of an inch	=	

(*d*) Notice that some of the inches upon your rule are divided into ten parts (tenths).

Count how many tenths of an inch there are in two inches and three-tenths ($2\frac{3}{10}$).

Measure the length of this page in inches and tenths of an inch.

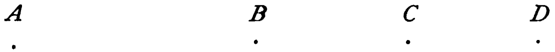
> Suppose you wished to put down that something you had measured was four inches and two-tenths long, you could write it in words, or you could write it in figures in this way: $4\frac{2}{10}$ inches. A still better way is to separate the tenths from the inches by means of a dot, thus: 4·2 inches.

This method of writing down numbers and fractions is known as the *Decimal System.*

(*e*) Write the following numbers down in figures with a dot to separate the tenths from the whole numbers:

One inch and four-tenths =
Two inches and six-tenths =
Eight inches and no tenths =
No inches and five-tenths =

(*f*) Measure the distances between the dots *AB*, *BC*, and *CD* below, in inches and tenths.

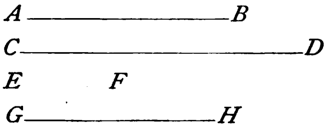


Length *AB* =
" *BC* =
" *CD* =
Total

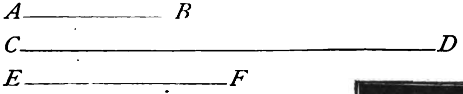
Now measure the distance *AD*, and see whether it agrees with the total you have found by addition.

Length *AD* =

In a similar way find the total length of the following lines:



(*g*) Measure the lengths of the lines *AB*, *CD*, *EF*, and subtract the total length *AB*+*EF* from the length *CD*.



2. Metric Measures of Length.

(*a*) Notice the divisions on the edge of a rule marked like the decimetre scale of Fig. 1.

The smallest divisions upon the edge of a scale or rule of this kind, known as a metric scale, are called *millimetres*—written shortly *mm.*—and one thousand of them make a *metre*.

The numbered divisions on the metric scale in Fig. 1 are called *centimetres*—written *cm.*—and one hundred of them make a metre.

Ten centimetres make one *decimetre* (*dm.*), and ten decimetres make one *metre* (*m.*).

Count and write down the number of millimetres in a centimetre.

(*b*) You should now be able to fill up the blanks in the following table, which you should enter in your note-book.

Metric Measures of Length.

1 cm. = …… mm.
1 dm. = …… cm. = …… mm.
1 m. = …… dm. = …… cm. = …… mm.
1 kilometre = 1,000 metres.

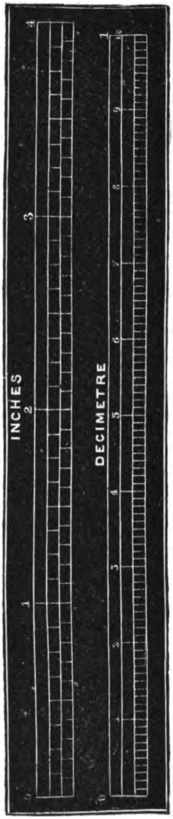


FIG. 1.—Comparison of inches and centimetres.

(*c*) Find out how many millimetres there are in the length this page, and also in the length of this line of print.

Measure the length of this page in centimetres.

Since a millimetre is one-tenth of a centimetre, the number of millimetres over may be written after a dot, as with tenths of inches. You know now that the number of tenth ($\frac{1}{10}$) parts of anything is shown by the first figure to the right of a decimal point. In a similar way, the number of hundredth ($\frac{1}{100}$) parts is expressed by the second figure to the right, and thousandths ($\frac{1}{1000}$) by the third figure, and so on. A length of:

Metres.	Decimetres. (Tenths of a Metre.)	Centimetres. (Hundredths of a Metre.)	Millimetres. (Thousandths of a Metre.)
2	6	4	8

may therefore be written in this way:

2·648 metres.

(*d*) Write the following numbers in metres and fractions of a metre in a similar manner to that shown in the example:

Metres.	Decimetres.	Centimetres.	Millimetres.	
6	8	3	7	= ...... m.
10	5	6	9	= ...... m.
0	3	7	4	= ...... m.
0	0	2	1	= ...... m.

3. Comparison of British and Metric Scales.

It is most important that the student should possess at least a rough idea of the comparative dimensions of British and Metric Measures of Length, so that if he sees a length expressed in one system, he can form a general notion as to what it would be in the other system. The following exercises have been arranged for this purpose:

Metric Equivalents of British Measures of Length.

(*a*) Carefully open a pair of dividers until the ends of the legs are exactly an inch apart. Place the ends upon the metric scale, and notice how many millimetres are included between them (Fig. 2). Write in your note-book the number of millimetres, and the number of centimetres and tenths of a centimetre, there are in an inch.

Also draw a line one inch long, and another line one centimetre long under it, writing the name against each.

(*b*) Find how many centimetres and tenths of a centimetre there are in the length of a foot on your scale. Thence calculate the number of centimetres in one inch.

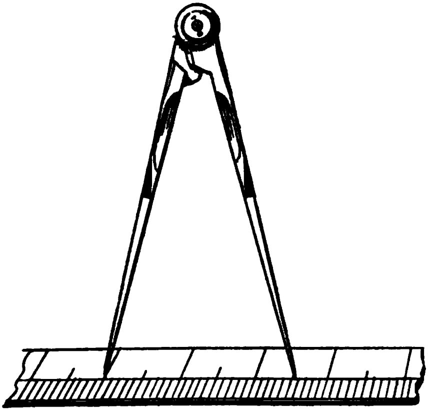


FIG. 2.—The points of the dividers are separated by 1 inch. 25·4 millimetres are equal to the length of an inch.

If you possess a tape measure, or a metre rule divided into inches and centimetres, notice and state how many centimetres there are in a yard.

In the absence of a long metric rule, procure a blind lath about four feet long, and draw a line straight across it near one end. Taking this line as the starting-point, divide one edge of the lath into 36 inches, that is, one yard. Divide the other edge into centimetres, making each tenth division longer, to indicate the decimetres. Notice and state how many centimetres are equivalent to one yard.

(*c*) You will now be able to fill up the blanks in the following table:

1 inch = ... cm. = ... mm
1 foot = ... dm. = ... cm. = ... mm.
1 yard = ... dm. = ... cm. = ... mm.
1 mile = 1609 metres = 1·609 kilometres.

British Equivalents of Metric Measures of Length.

(*d*) Find by observation of the divisions upon your scale, or by means of dividers as in Exercise 3 (*a*), the number of tenths of an inch there are in 1 centimetre. Write in your note-book the fraction of an inch equivalent to 1 cm. and to 1 mm.

Observe upon your scale, or determine by measurement, the number of inches equivalent to 1 decimetre. Draw a line 1 dm. long, and divide it into inches and tenths of inches.

Find the number of inches equivalent to 1 metre.

(*e*) You should now be able to fill up the blanks in the following table:

1 millimetre = ... inch.
1 centimetre = ... inch.
1 decimetre = ... inches.
1 metre = ... inches.
1 metre = ... feet.
1 metre = ... yards.
1 kilometre = 0·621 mile.

4. The Conversion of Units.

It is often necessary to convert British into Metric measures and *vice versa*. This is merely a matter of simple multiplication. Thus:

Convert 12·7 inches into centimetres.
Since 1 inch = 2·54 cms.
Therefore 12·7 inches = 2·54 × 12·7 cms.
= 32·258 cms.

The following examples, which need not be worked in the laboratory, will serve to make the student familiar with the use of conversion factors:

(*a*) If a man walk 12 miles in 3 hours, how many kilometres would he walk in the same time?

(*b*) If the length of a certain road is 1963 metres, what is the length in feet?

(*c*) A merchant wishes to order 220 feet length of a certain material from a continental firm which sells the material in metres. How many metres should he order?

(*d*) Convert (i.) 17·6 inches into centimetres.
(ii.) 2 ft. 3 in. into decimetres.
(iii.) 22 yds. 7 ft. 5 in. into metres.

(*e*) Convert (i.) 30 cms. into inches.
(ii.) 5·7 dms. into feet.
(iii.) 11·354 metres into yards.

(*f*) Measure the distance in centimetres between the two dots below, and also the distance of each dot from the bottom edge, and from the outside edge, of this page.

•   •

(*g*) Repeat the Exercise, making the measures in inches. Bring the results to centimetres (1 inch = 2·54 cm.), and compare the answers thus calculated with those found by direct measurement.

5. Average of Measurements.

(*a*) Measure the distance round your wrist with a strip of paper or a piece of string. Make several measurements and record each one in the form shown below; then find the mean or average, as is done in finding averages in cricket, by adding up the results, and dividing by the number of separate measures.

Wrist Measurement

1st measure =
2nd „ =
3rd „ =
Total - - -
Mean - - -

Whenever you make measures of any kind, the *mean* or *average* result should be obtained in this way.

(*b*) Measure the length and width of this book in two or three places, and find the mean length and width.

(*c*) Calculate the mean ratio of the width of this book to its length, by dividing the mean length by the mean width.

6. Ratio of the Diameter of a Circle to its Circumference.

(*a*) Wrap a strip of paper closely round a wooden cylinder (Fig. 3), and make a small hole with a pin at a place where the paper overlaps. Unroll the paper and measure the distance between the two holes. This gives you the distance round the cylinder, that is, its *circumference.*

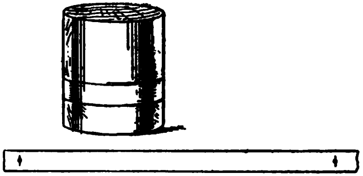


FIG. 3.—Measurement of the circumference of a cylinder.

Measure the circumference in another way by winding a piece of thread round the cylinder several times side by side, and dividing the total length of the thread by the number of coils.

Measure the diameter of the cylinder in several places by applying the rule to one end and taking the greatest distance across. Then find how many times the circumference is greater than the diameter.

CIRCUMFERENCE.	MEAN DIAMETER.	$\frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$.
		Mean.

Repeat the process at least three times, and take the mean of the results in the third column.

(*b*) Repeat the preceding exercise with cylinders of different sizes, such, for instance, as a tin canister, or a short piece of wide glass tubing, or a round bottle. Make the measurements in centimetres, and again find the relation between the circumference and the diameter.

> The circumference of every circle is 3$\frac{1}{7}$ (that is, $\frac{22}{7}$) or 3·1416 times the diameter. This number is represented by the Greek letter π. Your measures will not, of course, give the number exactly.

(*c*) Measure by the British scale the diameter of a halfpenny, taking care to measure the greatest distance across. Afterwards calculate the length of the circumference by multiplying the length of the diameter by 3$\frac{1}{7}$, or $\frac{22}{7}$, or by the decimal fraction 3·1416.

(*d*) Measure the diameter of a penny, and from it calculate the length of the circumference.

> (i.) Calculate the length of the circumference of a bicycle wheel 26 inches in diameter.
>
> (ii.) A bicycle wheel, 28 inches in diameter, is being driven at the rate of 80 revolutions per minute. If this rate is sustained, how far will the machine travel in an hour?

(*e*) To measure the diameter of a ball an indirect method must be used, because of the difficulty of measuring the diameter of a sphere directly with a scale.

Procure two rectangular wooden blocks larger across than the sphere. Push the two blocks against a third, or against any upright surface, so as to keep them parallel, and then place the wooden sphere between them and measure its diameter, as in Fig. 4. The scale is there shown lying flat on the blocks, and if it is a thin metal rule it can be used in this way. But if a wooden scale is used it should be placed on the blocks "edge

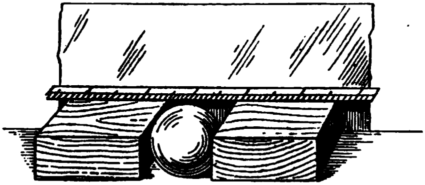


FIG. 4.—Measurement of the diameter of a sphere.

on," as in Fig. 5. Make measures with the sphere in different positions, and find the mean or average result. Record your results thus :

Measurement of Sphere between Blocks.

1st measure, - - - - - ...... cm.
2nd " - - - - - ...... cm.
3rd " - - - - - ...... cm.
4th " - - - - - ...... cm.

Total, - - - -

Mean Diameter, - - cm.

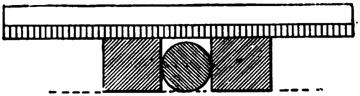


FIG. 5.—How to use a scale "edge on" to prevent inaccurate measurement.

7. Measurement of the Length of Curved Lines.

All curved lines can be regarded as made up of a very large number of small straight lines. The larger the number of small straight lines into which we suppose the line to be divided, the more nearly shall we get to the actual length of the curved line. If we could take an infinite number of such small straight lines, there would be no difference between the sum of their lengths and the length of the curved line. Fig. 6 will help you to understand these statements.

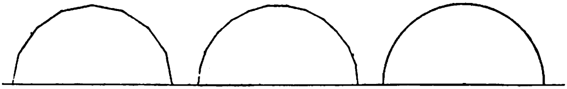


FIG. 6.—To illustrate how a curve may be considered to be made up of an infinite number of short lines.

(*a*) Draw (1) a circle of 1·5 cm. radius ; (2) The quadrant of a circle of 4 cm. radius. Measure the lengths of these curves by the help of dividers as follows: Make a mark on the curve with a fine pointed pencil as a starting point. Procure a fine pair of dividers and open them a small distance, using your scale to determine this distance: 3 mm. will be a convenient amount for the ends to be apart. Starting from the mark, place one leg of the dividers upon it, and see that it does not slip; then bring the other leg up till it is exactly on the curve. Put down 1 on a piece of paper, or keep count in your mind. Now keeping the second leg of the dividers firm, bring round the first until it is exactly on the curve and fix it there. Put down 2 on the piece of paper. Repeat this process until you have passed right round the curve. The number of times you have moved the dividers, multiplied by 3 mm., will give you the length of the curve in mm.

(*b*) Use a fine piece of thread instead of the dividers, and measure the length of the same curves with it.

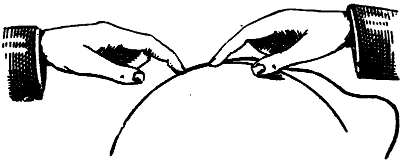


FIG. 7.—Measurement of a curved line with thread.

Starting at the mark as before, put one end of the thread upon it and hold it there with the nail of the first finger of your left hand. Make the thread coincide as nearly as you can with a small part of the curve, and place the nail of the first finger of your right hand upon it. Now release your left-hand finger and carefully place it at the point where your right-hand finger is held; then, using your right hand, go on to make some more of the string exactly coincide with another small length of curve. Repeat this until you have completed the whole curve. Measure the length of thread with your mm. scale. Compare the result with that obtained using the dividers.

(*c*) Stick a number of pins at close intervals along a curved line the ends of which meet; wrap cotton round these pins, and when the cotton overlaps cut it; measure the cut piece of cotton.

(*d*) Procure a tracing wheel, such as is used by dressmakers. Or, obtain a small toothed wheel with sharp teeth from a disused watch or clock, and mount it with a thin nail as an axle upon a strip of wood. Run the wheel across a page of your note-book, and measure the distance in millimetres between twenty of the impressions of the teeth. By dividing the result by twenty you can find the distance between two successive teeth. Repeat the determination. Write in your note-book :

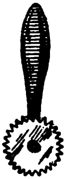


FIG. 8.—A tracing wheel for measuring curved lines.

Distance between twenty impressions, - - - ...... mm.
Therefore, distance between successive teeth, - - ...... mm.

Now determine the length of a curved line by running the tracing wheel along the line and counting the number of impressions it makes. This, multiplied by the distance between two teeth, gives the length of the curve.

(*e*) Obtain and examine an instrument called an *Opisometer*. A small wheel with fine teeth fits upon a screw of fine pitch which forms an axis, and as the wheel is turned it moves towards one end of the axis. Turn the wheel until it reaches one end of the screw. Make a mark upon it at the bottom. Place the mark at the starting point on the curve you wish to measure, and carefully cause the wheel to travel along the whole length of the curve. When it has completed the journey along the curve, make a mark on the rim of the wheel touching the end of the curve and lift the wheel off. Place this last mark over some division on your millimetre scale, and run the wheel along the scale in the opposite direction to which it formerly turned, so making it retrace its course along the screw axis. When it arrives at the end of the axis notice the division on the mm. scale. Record the length, and compare it with those obtained by the string and dividers.

(*f*) Draw a circle in your note-book by means of a pair of compasses. Measure the length of the circumference (*a*) with thread, (*b*) by a tracing wheel, (*c*) with an opisometer. Then measure the diameter and calculate the circumference. Enter your results as indicated below :

Length of circumference measured with thread, - - ......
" " " " tracing wheel, ...
" " " " opisometer, - ......
" " calculated, - - - - ......

(*g*) Draw another circle. Measure the circumference very accurately by the method you consider the most exact. Calculate the length of the diameter from your measures. Then measure the diameter, and compare the calculated with the observed result.

8. Fractional Scales.

(*a*) Measure the length of your table or desk and also the breadth. Record both measures in your note-book.

> If you wished to show this length by means of a line drawn upon this page, you could not do so without reducing the length in a certain proportion. Suppose you decide to let the length of one inch upon this page represent one foot length of your desk, then by drawing a line of the same number of inches and parts of inches as your desk measures feet, and parts of a foot, you will show the length of the desk on a scale of one inch to a foot, that is, $\frac{1}{12}$th the true scale.

Using your measures of length and breadth, make a *plan* showing your table or desk on a scale of one-twelfth.

(*b*) Taking one inch to represent one foot, that is, on a scale of $\frac{1}{12}$, draw a line a yard long.

Draw a line one pole long, on a scale of one inch to a yard, that is, $\frac{1}{36}$.

Taking $\frac{1}{10}$ of an inch to represent one yard, that is, a scale of $\frac{1}{360}$, draw a line 1 chain (22 yards) long.

(*c*) With $\frac{1}{10}$ inch to represent a yard, make two marks upon a piece of tape, so that the distance between them represents one mile. Make a mark at each furlong from the starting point. Also mark off from the starting point distances representing one pole and one chain. These marks will give you a clear idea of the proportions in the following table:

Long Measure.

12 Inches (in.)	make	1 Foot (ft.).
3 Feet	,,	1 Yard (yd.).
5½ Yards	,,	1 Pole, Rod, or Perch.
40 Poles (220 yds.)	,,	1 Furlong (fur.)
8 Furlongs (1760 yds.)	,,	1 Mile.
22 Yards = 100 links	,,	1 Chain (ch.).

9. Calipers.

It is sometimes inconvenient, or impossible, to use the steel scale directly for the measurement of lengths. In such cases it is often an advantage to use simple instruments, called calipers, which are often made of the form shown in Fig. 9. One pair of legs is for inside, the other pair for outside, measurements. In using the former the jaws are opened until they each lightly touch the insides of the cavity the diameter of which is required. Similarly, in using the outside calipers, the distance between the jaws is adjusted until the object passes between them, just touching them. The distance between the points of the jaws of the calipers is then read by placing the calipers on the divisions of the scale used to measure lengths.

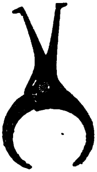


FIG. 9.—Calipers.

(*a*) Using a pair of inside calipers, and your metric scale, measure the inside diameter of a glass cylinder and of a piece of large glass tubing, in millimetres and inches.

(*b*) With the outside calipers, and your metric scale, measure the diameter of a penny and halfpenny, and compare with the results obtained in Exercise 6 (*c*).

By Calipers.

Diameter of penny = ... cm.
Diameter of halfpenny = ... cm.

By Direct Measurement.

Diameter of penny = ... cm.
Diameter of halfpenny = ... cm.

(*c*) Similarly measure the diameters of the cylinder of Exercise 6 (*a*) and the sphere of Exercise 6 (*e*), using the outside calipers instead of the method there adopted.

Measurement of Diameter of Cylinder.

	(*a*) WITH SCALE.		(*b*) WITH CALIPERS.
1st measure, -	...... cm.		...... cm.
2nd „ -	...... cm.		...... cm.
3rd „ -	...... cm.		...... cm.
4th „ -	...... cm.		...... cm.
Total, -		Total,	
Mean, -		Mean,	

Measurement of Diameter of Sphere.

	(*a*) BETWEEN BLOCKS.		(*b*) WITH CALIPERS.
1st measure, -	...... cm.		...... cm.
2nd „ -	...... cm		...... cm.
3rd „ -	...... cm.		...... cm.
4th „ -	...... cm.		...... cm.
Total, -		Total,	
Mean, -		Mean,	

10. The Diagonal Scale.

In taking measurements by means of an ordinary steel scale it often happens that the length which is being measured is not an exact number of divisions on the scale. The length is a certain number of divisions and a fraction of a division over. In making exact measurements you must be able to tell what fraction precisely is over in every case. One plan sometimes adopted is what is known

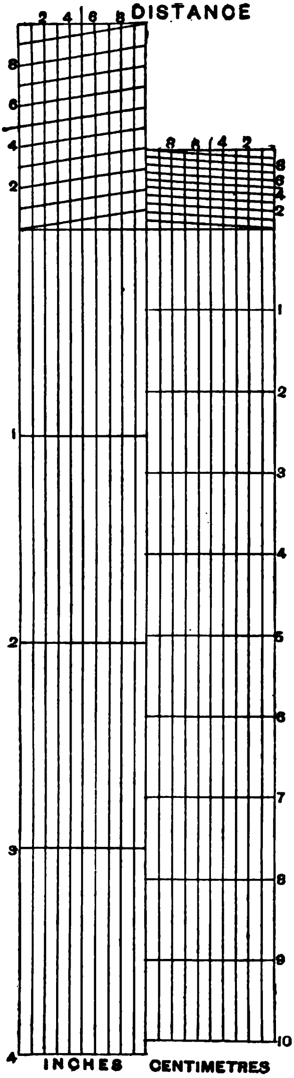


FIG. 10.—Diagonal scales. From Aldous's *Course of Physics* (Macmillan).

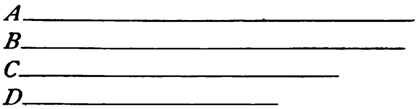
as the diagonal scale. Such a scale is easily understood by referring to Fig. 10. The left outside edge of the scale there shown is divided into inches, which are numbered. The scale is continued above the zero division for another inch, which is divided into tenths, some of which are numbered. Similarly the breadth of the scale is divided into tenths by the vertical lines drawn parallel to the outer edge. Diagonal lines are now drawn; the first joins the tenth marked *zero* on the left hand outside edge to the *first* tenth division on the right hand outside edge; the second diagonal line joins the *first* tenth division on the left outside edge to the *second* tenth division on the right outside edge, and similarly all the way along as shown in Fig. 10.

(*a*) Construct a diagonal scal following the plan just described.

Take off the number 3·75 inches on the diagonal scale. To do this place one leg of your dividers on the horizontal line marked 3 (which is the *first* digit of your number) at the place where the fifth (that is the *third* digit of your number) vertical line intersects it. Open out the dividers until the other leg just reaches the intersection of the fifth vertical with the seventh (that is the *second* digit of your number) diagonal line. The length between the points of the dividers will be the length 3·75 inches.

Similarly take off distances 2·81 in., 4·97 in., 1·86 in.

(*b*) Find the exact length of the following lines by means of the diagonal scale you have constructed, or by using the scale in Fig. 10:



Find the lengths of the same lines in centimetres, by means of the right-hand diagonal scale in Fig. 10.

The principle of the diagonal scale is used in the following method of determining the internal diameter of a tube:

(*c*) Take a strip of squared paper, divided into centimetres and millimetres, and cut from it a right-angled triangle, with a base 10 cm. long, and the perpendicular 1 cm. long (Fig. 11). Using a glass tube, press the acute angle into the tube till it will go no farther, then the diameter will be given, in cms., by one-tenth of the length (in cms.) of that part of the base which has been introduced into the tube.

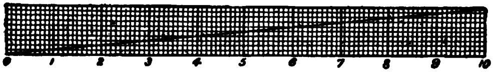


Fig. 11.—Construction of gauge to measure the internal diameter of a tube.

11. Construction and Use of a Vernier.

(*a*) Construct a vernier as follows: Draw a scale of 30 equal parts. It will be convenient to mark off tenths of an inch from your steel scale. Lengthen every fifth and every tenth part as shown in Fig. 12. Suppose the vernier is required to read to the tenth part of these equal divisions, you must first divide nine of the divisions of the scale into ten equal parts by an ordinary geometrical method, as shown in Fig. 12. The ends of a pair of compasses are opened until they are 10 divisions apart. One leg of the compass is placed at *C*, and a small arc is drawn which cuts the line *AB* at *D*. The points *CD* are joined, and the line *CD* is divided into ten of the divisions of the scale. Perpendiculars are drawn from these ten divisions, and they divide the length of nine divisions *AC* into ten equal parts. It is quite clear, therefore, that each of the divisions so obtained is one tenth shorter than the scale divisions.

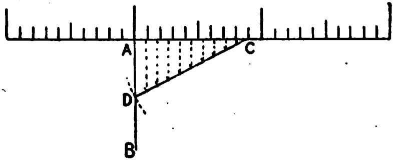


FIG. 12.—Construction of a vernier to read to one-tenth of a division on the scale.

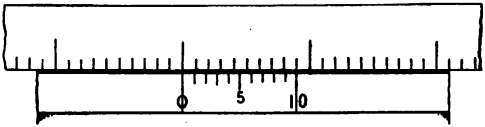


FIG. 13.—A vernier upon which 10 divisions are equal to 9 on the scale.

Now draw, in ink, short lines from each of the divisions so obtained, and then rub out the construction lines, so that the final result is as shown in Fig. 13.

This small scale will be the required vernier.

> A vernier can also be obtained by dividing 11 scale divisions into 10 equal parts, but in this case the divisions of the vernier must be numbered in the opposite way. One advantage about this plan is. that the vernier scale is more open, that is, the divisions are farther apart.

(*b*) Construct a vernier scale to read to $\frac{1}{10}$, dividing eleven-scale divisions into ten parts, and numbering the vernier scale in the opposite way to that in Exercise 11 (*a*).

General Rule for Construction of Vernier Scales.

To read to the *n*th part of a scale division, *n* divisions of the vernier must equal $n+1$ or $n-1$ divisions on the scale, according as the vernier and scale read in opposite or similar directions.

If, therefore, it is desired to construct a vernier which will read to the 20th part of one of the scale divisions, 20 divisions on the vernier must be equal in length to 21 on the scale, if the vernier is to read in the opposite direction to the numbering on the scale, and to 19 scale divisions if it is to read (as is usually the case) in the same direction as the scale numbering.

Use of a Vernier.

In using a vernier the student must look along the divisions upon it until he sees one which coincides with a scale division. If the vernier reads to the tenth of the length between two scale divisions, and the first division upon it coincides with a scale division, then the zero of the vernier is $\frac{1}{10}$th of a space past the division near it; if the second vernier division coincides, the zero is $\frac{2}{10}$ths

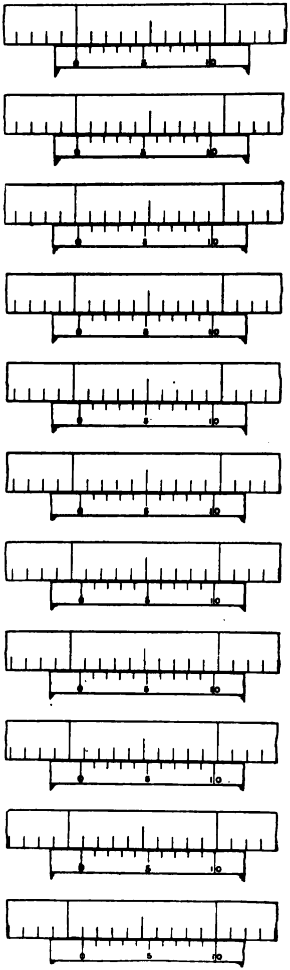


FIG. 14.—Positions of a vernier for successive advances of one-tenth of a scale division.

past, and so on for any of the ten vernier divisions. Fig. 14 shows the position of the vernier for each tenth of a division in advance of the first position, beginning at the top.

(*c*) Using a model vernier or a boxwood scale with vernier scale attached, practice vernier readings by pushing the vernier into a series of different positions along the scale. Draw diagrams showing the relative positions of the vernier and scale divisions in two cases, and write against each the reading it shows.

12. The Slide Calipers.

The construction of the slide calipers is easily understood by a reference to Fig. 15.

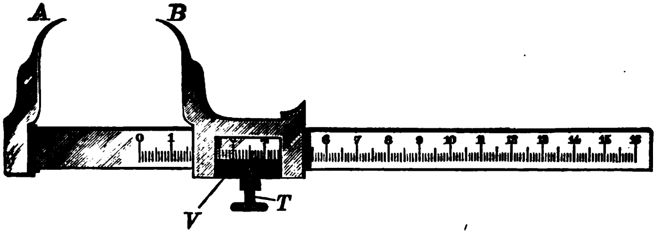


FIG. 15.—Slide Calipers. *A* is a fixed jaw and *B* a movable one, which can be fixed by means of the screw *T*. Upon this slide is a vernier *V*.

(*a*) Compare the slide calipers supplied you with Fig. 15, and notice there are two metal jaws, one of which is fixed at the end, while the other slides along a carefully graduated scale. This scale is sometimes divided like your metre scale, and sometimes into inches and tenths. The movable jaw can be fixed in any position by a screw. The two inside faces of the jaws are at right angles to the scale, and where the movable jaw moves along the scale, it is made into a fine knife edge to enable its exact position to be read easily.

Using the slide calipers, measure the diameter of the cylinder and sphere of Exercises 6 (*a*), and 6 (*e*). Compare the results with those already obtained.

(*b*) Sometimes a vernier is attached to the slide calipers, which of course adds much to the accuracy of the readings. If there is such an instrument in the laboratory, repeat the last experiment with it, and compare the results.

13. Principle of Micrometer Screw.

(*a*) Obtain a large screw or bolt and a nut into which it fits. (Fig. 16.) Scratch a line along the tops of the screw thread from one end of the screw to the other in the direction of the axis.

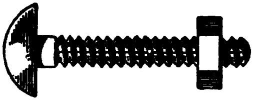


FIG. 16.—Bolt and nut to illustrate the principle of the micrometer screw.

Place your rule along the screw and see how many of the lengths between successive scratches are equal to the length of an inch; or measure the distances between say 12 or 15 of the marks. You can thus find the distance between two successive marks. Record this distance in your note-book.

Put the nut upon the screw and turn it until it reaches one of the marks upon the thread. Make a line upon the nut at the point where the marked thread touches it. Turn the screw, and notice that one complete turn brings another scratch on the thread to the side of the line upon the nut.

> You thus see that one complete turn advances the screw by an amount equal to the distance between two successive threads. You know what this distance is, and can therefore write down the distance through which the screw advances for any number of turns.

(*b*) Find (i) by calculation, using the value you have found for the distance between successive threads, (ii) by direct measurement, the amount of advance of the screw with reference to the nut when it is given 7, 8½, 10, 11½, and 14 turns.

14. Construction of a Model Screw Thread.

(*a*) Cut out of paper a right-angled triangle such as *ABC* (Fig. 17) and wind it round a lead pencil (Fig. 18). The slant side of the triangle forms a spiral upon the pencil, similar in appearance to the thread of a screw. If the inclination of your triangle is small, the threads appear close together, and if it is large they occur farther apart. Mark where the end of the paper *C*, touches the base of the triangle and draw a line, perpendicular to the base, from this point to the slant side.

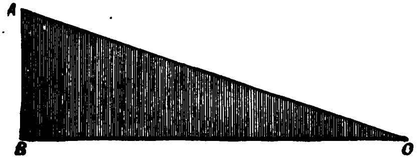


FIG. 17.—A right-angled triangle of paper to form a screw thread.

The small triangle *CDE* (Fig. 18) thus formed is similar to the large one, and it represents one turn of the screw-thread. You see therefore that:

Height of triangle *CDE* represents distance between threads;

Base of triangle represents circumference of screw.

The shortest distance between two points on screw threads next to one another is represented by the height *DE* of the triangle *CDE*. The angle *ECD* determines this height; and the height itself is known as the *pitch* of the screw.

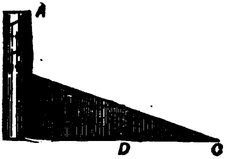


FIG. 18.—Screw thread formed by wrapping a right-angled triangle round a pencil.

Consequently, if the pencil is advanced by turning it once round, that is through one turn, the screw moves forward by the distance between two threads.

Now if a head is attached to the screw and the circumference of it is divided into say 100 parts, then when the head is turned round the hundredth part of a turn, the screw is advanced through one hundredth of the distance between two threads. This is the principle of the micrometer screw gauge.

15. The Micrometer Screw Gauge.

(*a*) Compare the instrument supplied you with Fig. 19. Notice the carefully cut screw. Unscrew the head, turning it through *one* complete revolution; notice that the screw is advanced or put back through *one* long division of the linear scale at *F*.

Similarly, turn the head through two, three, or any convenient number of turns; notice that the screw is moved through a corresponding number of long divisions on the scale at *F*.

The distance between the threads, or the pitch of the screw, is some definite unit of length. What is it in your gauge?

Into how many parts is the movable head itself graduated? If you move the head round one of these divisions, through what fraction of a division on the linear scale at *F* is the screw moved?

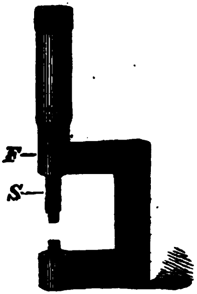


FIG. 19.—A Micrometer screw gauge. *F* is a linear scale for showing the number of whole or half turns of the screw *S*.

(*b*) Test the *zero reading* of your screw gauge. Slowly and carefully screw up until the movable and fixed jaws *just* touch. *Very great care must be taken not to injure the screw thread by screwing it up too hard.* The division marked 0 on the graduated edge of the movable head should be exactly over the projecting line of the horizontal scale. If this is not so, make several experiments, writing the error in each case in your notebook, and, unless you are able to properly adjust the screw to the true zero, always take the *mean* of these observations of error into account in any measurements you make with the screw gauge.

(*c*) Use the micrometer screw gauge to measure the diameters of several pieces of glass tubing of different sizes.

(*d*) Measure the diameters of four wires *A*, *B*, *C*, *D* by means of the screw gauge. Take several readings at different places along each wire, and then find the mean reading for each wire. At *each* place on the wires, readings should be taken at right angles to one another and the mean taken, as the wire will probably not be quite round.

	A	*B*	*C*	*D*
1st reading,	......	......	......	......
2nd „	......	......	......	......
3rd „	......	......	......	......
4th „	......	......	......	......
Total,				
Mean,				

(*e*) Measure the diameters of the wires used in the preceding exercise in the following way: Wind the wire carefully round a lead pencil or glass tube, taking care that each turn of the wire touches the previous one at all points. Make 20 turns, and measure the distance from the outside of the first turn to the outside of the twentieth turn by means of a millimetre scale. Divide this measurement by 20, and so obtain the diameter of the wire. Repeat the experiment with varying numbers of turns of wire.

Length of 20 turns of wire, - - ...... mm.
∴ Diameter of wire - - - - - ...... mm.

16. The Spherometer.

The principle of the construction of the spherometer is precisely the same as that of a micrometer screw gauge.

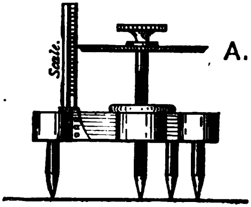


FIG. 20.—Spherometer with all its feet on a plane surface. From Earl's *Physical Measurements*, (Macmillan).

(*a*) Carefully place a spherometer upon a piece of plate glass, and turn the head *A* slowly until the end of the central screw touches the glass, as indicated in Fig. 20. (If you turn the screw too far you will find that the instrument will rock upon the glass plate instead of standing firmly upon it.) Notice the division of the vertical scale in a line with the disc upon the screw. Turn the disc through one complete revolution, and again notice the reading on the scale. Turn it through two or three more revolutions and observe in each case the rise of the disc with reference to the scale.

(*b*) Select a piece of plate glass, or any other object of which you know the exact thickness, and place it on a flat base (Fig. 21). Turn the disc of the spherometer until the point of the screw just rests upon the object used, taking care to count the number of complete revolutions necessary to accomplish this. By dividing the known thickness of the object used by the number of turns and fraction of a turn observed, the distance through which the point of the screw moves for one revolution is found. Determine in this way the distance through which the point of the screw moves for one turn of the disc, that is, the pitch of the screw of the spherometer.

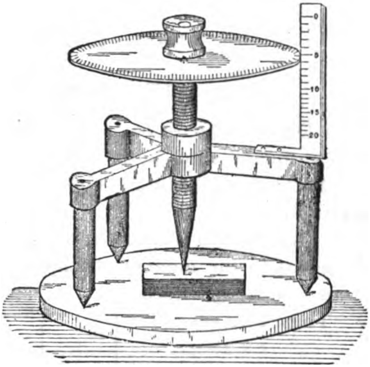


FIG. 21.—Use of a spherometer. From Nichols's *Junior Course of General Physics* (Macmillan).

If the point of the screw of a spherometer moves through one millimetre for one revolution of the disc, through what distances does it move when the disc is turned through one-tenth of a revolution and through one-hundredth of a revolution?

(*c*) Count the number of divisions upon the disc of the spherometer. Through what distance does the point of the screw move when the disc is turned through one division?

Measure the thickness of a halfpenny by means of the spherometer. Measure also the thickness of a piece of glass, and of a piece of cardboard.

CHAPTER II.

MEASUREMENT OF AREA.

17. Units of Area.

(*a*) Draw a square each side of which is one inch long, and also a square with sides two inches long. Divide the latter into four square inches. Then draw a square containing nine square inches.

Notice that in each of the cases the number of squares is equal to the number obtained by multiplying the length of a side by itself.

Thus $1 \times 1 = 1$;

$2 \times 2 = 4$;

$3 \times 3 = 9$.

Multiplying a number by itself is called *squaring* it, and the result is known as the *square* of the number. When a small 2 is written near the top to the right of a number in this way, 6^2, it means that the square of the number must be used; in fact, the small figure, which is called an *index*, tells you how many sixes must be multiplied together; thus, $6^2 = 6 \times 6 = 36$.

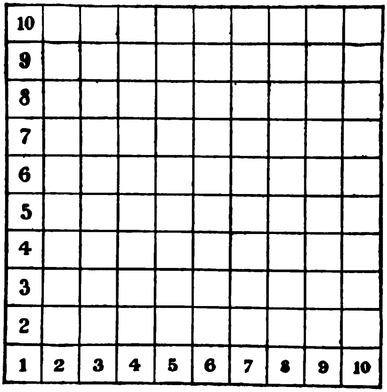


FIG. 22.—To illustrate how to divide a square decimetre into square centimetres.

(*b*) What numbers are the squares of 4, 5, and 6?

Find the number which, when multiplied by itself, produces 64. The number you find is the *square root* of 64.

Find the square roots of 100, 25, and 81.

(*c*) Draw a square of a decimetre side and divide two adjacent sides into centimetres. Draw lines through the divisions parallel to these sides and count how many square centimetres thus formed there are in a square decimetre.

Notice that you obtain the same result by counting the squares along the alternate sides and multiplying together.

(*d*) The preceding exercise has shown you that, to find the surface or area of a rectangle, that is, a four-sided right-angled figure, it is only necessary to multiply the length of the bottom or base by the height. Use this knowledge to find the number of square centimetres in the rectangles in Fig. 23.

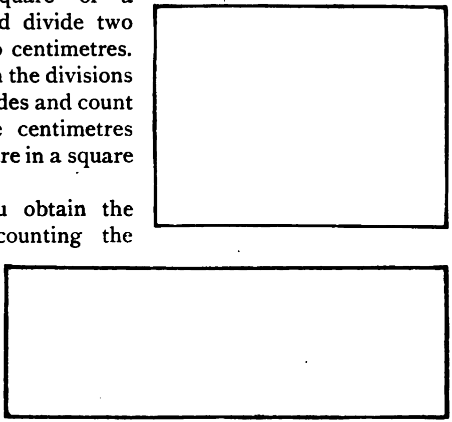


FIG. 23.—How many square centimetres are there in each of these rectangles?

18. Representation of Areas on a Reduced Scale.

(*a*) Taking one of the small squares upon the squared paper supplied you to represent 1 square inch, draw lines enclosing 144 of them, that is, 1 square foot, and also draw lines enclosing 1 square yard upon the same scale.

Taking one of the small squares on your squared paper to represent one acre, draw lines so as to enclose 640 acres, that is, one square mile.

(*b*) Upon a scale of $\frac{1}{16}$ of an inch to a yard, draw an area to represent an acre.

> To do this, first find two numbers, which, when multiplied together, make 4840, that is, the number of square yards in an acre. Then draw a rectangle having a base so many sixteenths of an inch long as are given by one of the numbers you find, and a height equal to the other number.

Divide the figure you draw into roods, and divide one of the roods into poles.

19. Comparison of British and Metric Units of Area.

(*a*) Fill up the blanks in the following table; not from memory, but by considering how many square inches there are in a square with sides each one foot long, and so on for other numbers:

British Square Measure.

...... Square Inches (sq. in.) make 1 Square Foot (sq. ft.)
...... „ Feet „ 1 „ Yard (sq. yd.)
...... „ Yards „ 1 „ Pole (sq. p.)
40 „ Poles „ 1 Rood (r.)
4 Roods „ 1 Acre (a.)
4840 Square Yards „ 1 Acre.
10 Square Chains „ 1 Acre.
640 Acres „ 1 Square Mile.

(*b*) Think what numbers are required to complete the following table, and enter the completed table in your note-book:

Metric Measures of Area.

...... square millimetres (sq. mm.) = 1 square centimetre (sq. cm.)
...... „ centimetres = 1 „ decimetre (sq. dm.)
...... „ decimetres = 1 „ metre (sq. m.)
100 „ metres = 1 are.

(*c*) Draw in your note-book a square inch and a square centimetre side by side, as in Fig. 24.

Draw a square of any convenient size; find the number of square centimetres in it. Then measure the length of a side in inches, and calculate the number of square inches there are in the figure. You will thus obtain the number of square inches equivalent to a certain number of square centimetres.

Use this proportion and your previous measures to find the number of square centimetres there are in a square inch, thus:

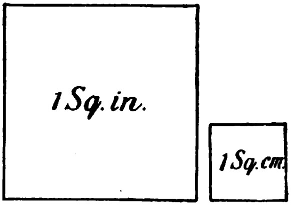


FIG. 24.—Comparative sizes of the square inch and square centimetre.

AREA IN SQUARE CENTIMETRES.	AREA IN SQUARE INCHES.	$\frac{\text{SQUARE CENTIMETRES}}{\text{SQUARE INCHES}}$.

It will be understood from Exercises 3(*b*), 3(*c*) that if measurements are made in either the Metric or British scales, they can be converted into the other system by a simple calculation. Before this calculation can be made, however, it is necessary to know the proportions which the units in one system bear to those in the other. These proportions have already been determined for units of length, and the relations between the units of area can be calculated from them. Thus, 1 inch has been found to be equal to 2·54 cm. 1 square inch is therefore equal to 2·54 × 2·54 sq. cms., that is, 6·451 sq. cm.

In a similar way, the proportion which other units of area bear to one another can be determined by squaring the numbers which represent the proportions between the units of length, that is, by squaring the numbers found in Exercises 3(*c*), 3(*e*). As your numbers may not be exactly correct, the accurate proportions are given in the following exercise, which should be worked at home.

(*d*) Copy the following tables in your note-book, and calculate the numbers to fill up the blank spaces.

British Units of Area to Metric.

1 square inch $=(2\cdot 54)^2$ sq. cm. = ... sq. cm.
1 square foot $=(3\cdot 05)^2$ sq. dm. = ... sq. dm.
1 square yard $=(0\cdot 91)^2$ sq. m. = ... sq. m.
1 square mile $=(1\cdot 61)^2$ sq. km. = ... sq. km.

Metric Units of Area to British.

1 square centimetre $=(0\cdot 39)^2$ sq. in. = ... sq. in.
1 square decimetre $=(3\cdot 94)^2$ sq. in. = ... sq. in.
1 square metre $=(3\cdot 28)^2$ sq. ft. = ... sq. ft.
1 square metre $=(1\cdot 09)^2$ sq. yd. = ... sq. yd.
1 are (100 sq. m.) $=(1\cdot 09)^2 \times 100$ sq. yd. = ... sq. yd.

(*e*) Draw a rectangle and determine its area in square centimetres. Calculate from the result the number of square inches in the rectangle.

(*f*) From the measures recorded in the first column of Exercise 19(*c*) calculate the numbers for the second column. Similarly, calculate the first column from the measures recorded in the second. Write in parallel columns in your note-book the observed and calculated results in both cases.

20. Determination of Areas by Squared Paper.

(*a*) Draw two parallel lines a short distance apart upon squared paper. Mark off equal lengths upon one of the lines and construct two parallelograms as in Fig. 25. (Every four-sided figure the opposite sides of which are parallel is a

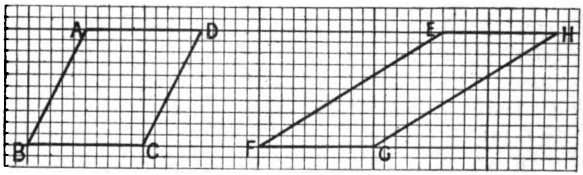


FIG. 25.—Parallelograms on equal bases and between the same parallels.

parallelogram.) Count the number of squares in each parallelogram, and estimate the fractions near the sides.

Number of squares in the parallelogram *ABCD* ....
" " " *EFGH* ....

The two parallelograms are between the same parallels, and on equal bases. What do your numbers teach you concerning the areas of such figures?

(*b*) Draw two parallel lines upon squared paper as in the preceding exercise, and draw between them two triangles having equal bases (Fig 26). Count the number of squares in each triangle as before.

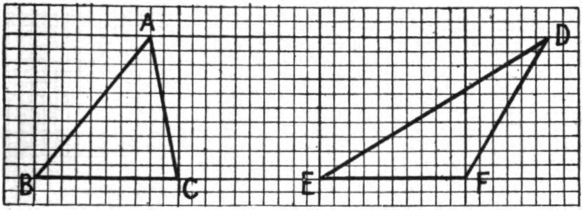


FIG. 26.—Triangles on equal bases and between the same parallels.

Number of squares in triangle *ABC* ....
" " " *DEF* ....

The triangles are upon equal bases, and between the same parallels. What do your numbers teach you as to their areas?

(*c*) Draw two right-angled triangles upon squared paper, and construct a square upon each side, as in Fig. 27. Count the number of squares, and estimate the parts of squares, in *ABDE*, and in *ACGF*, *BHIC* taken together.

	NUMBER OF SQUARES IN *ABDE*.	NUMBER OF SQUARES IN *ACGF*+*BHIC*.
1st triangle,		
2nd "		

What do your numbers teach you about the areas of the squares constructed upon the sides of a right-angled triangle?

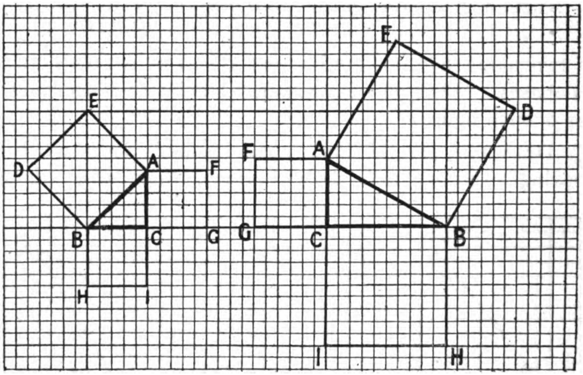


Fig. 27.—Squares on the sides of right-angled triangles.

(*d*) Using squared paper and the method employed in these exercises, determine the areas of a series of irregular figures, like those in Fig. 28. Then count up as before the number of

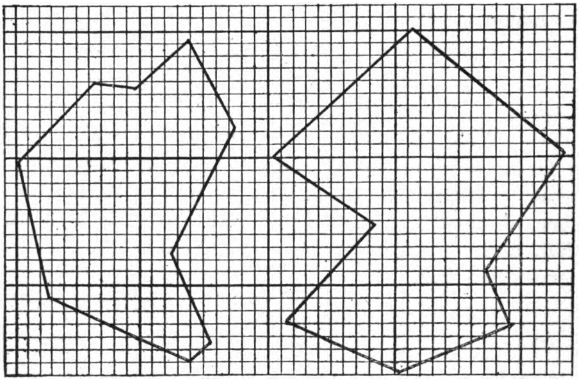


Fig. 28.—Areas of irregular figures determined by squared paper.

squares enclosed by each figure. Knowing the area of each small square, the area of the figure is determined.

21. Determination of Areas Geometrically and by Measurement.

(*a*) Cut out two cardboard parallelograms *ABCD*, *EFGH* (Fig. 29) and draw a line from *D* perpendicular to *BC*, and from *G* perpendicular to *EF*. Cut off the two triangles *DLC*, *GMF*, and place them so as to convert each parallelogram into a rectangle.

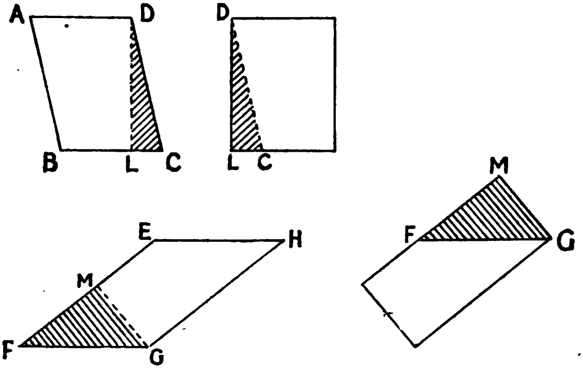


FIG. 29.—Every parallelogram can be considered as a rectangle.

Evidently the area of each complete figure is the same whether the triangle is in one position or the other. In other words, a parallelogram has the same area as a rectangle on the same base and having the same *altitude*, or perpendicular height. You can, in fact, consider every parallelogram as equivalent in area to an oblong or rectangle.

You have seen in Exercise 17 (*c*) that the area of a rectangle is equal to the base multiplied by the height. The same rule evidently holds good for determining the area of a parallelogram. It does not matter which side is considered as the base. Test this statement by means of the following exercise :

(*b*) Draw any parallelogram such as *ABCD* (Fig. 30), and find the area first by taking *DA* as base, and then by taking *CD* as base.

(*c*) Draw a parallelogram on paper or thin card, and then cut it in two from corner to corner. You have now two triangles, and by laying one on the other you will find that they fit or are equal in area (Fig. 31). Repeat the exercise with a parallelogram of different form.

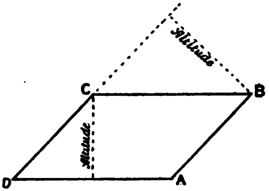


FIG. 30.—Determination of the area of a parallelogram.

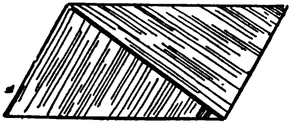


FIG. 31.—A parallelogram can be cut into two equal triangles.

You have seen that

Area of parallelogram = base × altitude.

But a triangle is half a parallelogram,

$$\text{Therefore area of triangle} = \frac{\text{base} \times \text{altitude}}{2}.$$

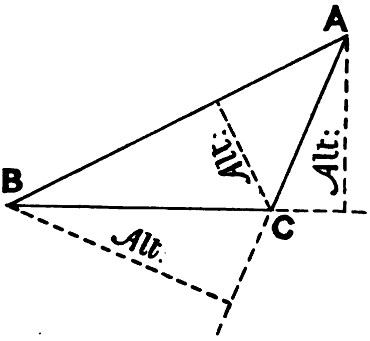


FIG. 32.—Determination of the area of a triangle.

(*d*) Draw any triangle as *ABC* (Fig. 32), and find its area by taking each side in turn as base.

The area of any surface bounded by straight lines can be obtained by dividing the surface into triangles and then adding up the areas of the triangles.

(*e*) Draw figures similar to those in Figs. 33 and 34, and find their areas by determining the total areas of the triangles into which each is divided. The same method can be applied to find the area of a plot of land bounded by straight lines.

In the case of figures bounded by irregular lines, the same method may often be adopted, and the error due to the departure from regularity may be estimated. Fig. 35 shows irregular areas divided into triangular and rectangular parts in this way.

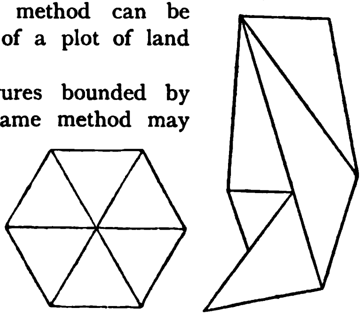


FIG. 33.—Determination of the areas of figures bounded by straight lines.

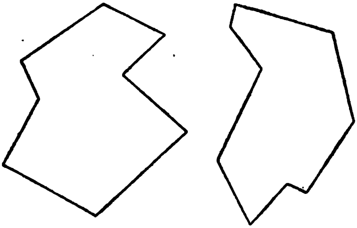


FIG. 34.—Figures for the determination of areas by division into triangles.

(*f*) Draw outlines of this kind upon cardboard or paper, and determine the areas in the manner indicated. Determine also the areas of the same figures by tracing the figures upon squared paper, and compare the two sets of results.

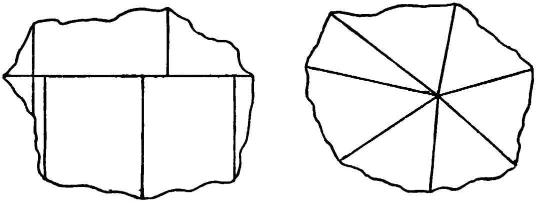


FIG. 35.—Determination of the areas of figures bounded by irregular lines.

22. Area of Circle.

(*a*) Cut out a circular disc of cardboard about four inches in diameter. Divide it into small triangles as in Fig. 36. The area of the circle could be found by determining the areas of all these triangles.

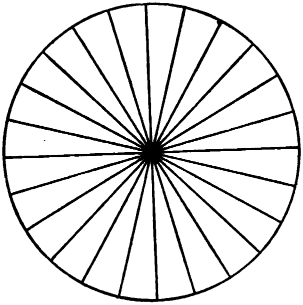


FIG. 36.—Circle divided into triangles.

Cut out the triangles and arrange them as in Fig. 37.

Find the area of this figure, regarding it as a parallelogram.

BD is the length of the radius of the circle, and *AB* is half the length of the circumference. Therefore, since the area of the figure equals that of the original circle, ***the area of a circle is equal to the radius multiplied by half the circumference.***

(It should be understood that this rule is only approximately proved by the method adopted.)

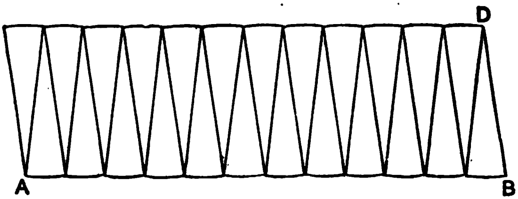


FIG. 37.—Figure formed by triangles cut from a circular disc.

You have found in Exercises 6(*c*), 6(*e*) that

$$\text{Circumference of circle} = \pi \times \text{diameter}$$
$$= \pi \times 2 \text{ radius}$$
$$= 3{\cdot}1416 \times 2 \text{ rad.}$$

The rule found by Exercise 22(*a*) may be simplified in the following way:

$$\text{Area of circle} = \frac{\text{circumference}}{2} \times \text{rad.}$$
$$= \frac{\pi \times 2 \text{ rad.}}{2} \times \text{rad.}$$
$$= \pi \times \text{rad.} \times \text{rad.}$$
$$= \pi \times \text{rad.}^2$$

Calculate the areas of a circular disc 12 inches in diameter, and of the floor of a circular hall 50 feet in diameter.

(*c*) Draw a circle upon squared paper, and determine its area. Find also the area of a square drawn upon the radius of the circle. Divide the former by the latter result, so as to determine the ratio between the area of a circle and the area of the square on the radius.

(*d*) Draw a circle upon squared paper, taking as radius the distance between 10 lines. Count the number of squares in the circle, estimating, as before, the fractions of squares. Now calculate the area from the formula, area of circle $= \pi \times \text{rad.}^2$, the radius being 10. You will thus be able to verify practically the accuracy of the formula given above.

23. Surface of a Sphere.

It is difficult to determine experimentally the area of the surface of a sphere, but the following device provides a very approximate illustration of the rule for calculating the area from the radius.

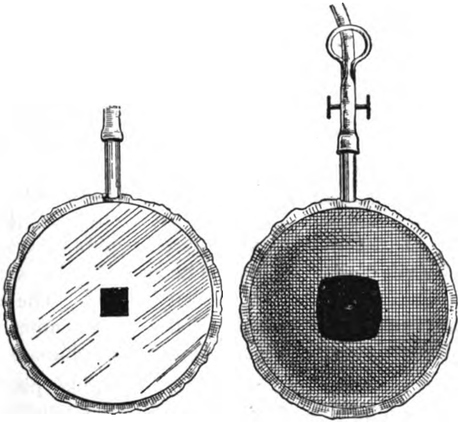


FIG. 38.—Device for indicating that the surface of a sphere has four times the area of the hemispherical section.

(*a*) Make or procure the apparatus shown in Fig. 38. To make the instrument, a narrow metal ring with a neck is obtained. Thin sheet india-rubber is fastened upon the ring, so as to form a hollow box with flexible ends. A square centimetre is drawn upon the india-rubber. A flat disc, which can be inflated into a sphere of the same diameter is thus obtained.

By means of a tube fastened upon the neck blow out the india-rubber until it takes the form of a ball. While it is stretched measure the area of the square marked upon it. It will be found approximately four times greater than it was originally, thus indicating that the surface of a sphere has four times the area of the hemispherical section or flat part obtained by cutting it into two halves.

(*b*) The area of the surface of a sphere may also be found in the following way: Procure an old tennis ball. Measure its diameter and then cut off the flannel covering; flatten each half section down on square paper and find the area. Add the two to get the area of the whole surface. Hence prove that the area of the sphere is four times that of a circle of the same diameter.

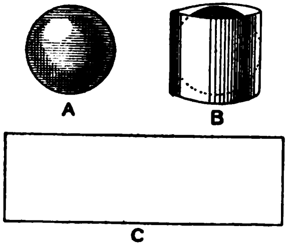


FIG. 39.—*A*, a sphere; *B*, sphere in circumscribing cylinder; *C*, circumscribing cylinder unrolled.

A fives ball may be similarly employed, the four coatings being cut off and their areas measured.

You have seen that the area of a circle is $3\frac{1}{7}$ times the square of the radius, or in brief symbols, $\pi \times r^2$, where $\pi = 3\frac{1}{7}$ and $r =$ the radius. The area of the surface of a sphere is 4 times this value, that is, $4\pi r^2$. The area of the curved surface of a cylindrical box of the same height and diameter as a sphere equals that of the sphere.

(*c*) Obtain a sphere, and cut out a rectangular piece of paper to go exactly round it, and having the same height. (Fig. 39.)

It will be understood that

Base of rectangle = circumference of sphere,
Height ,, = diameter of sphere,

and as the area of a rectangle = base × height,
the area in this case = circumference of sphere × diameter of sphere.

But $\text{circumference} = \pi \times \text{diameter}$

$= \pi \times 2 \text{ rad.}$

Therefore $\text{circumference} \times \text{diameter} = \pi \times 2 \text{ rad.} \times 2 \text{ rad.}$

$= 4 (\pi \times \text{rad.}^2).$

But this is the formula for determining the area of a sphere. *The area of the curved surface of a cylinder surrounding a sphere and of the same height is, therefore, exactly the same as the area of the sphere itself*, for both are equal to $4 (\pi \times \text{rad.}^2)$.

(*d*) Calculate the area of the surface of a globe 18 in. in diameter.

The rule used in the preceding exercise to find the area of a sphere can be expressed in different ways, thus :

(1) Area of sphere = circumference × diameter
(2) „ „ $= 3{\cdot}1416 \times$ square of diameter
(3) „ „ $= 4 (\pi \times \text{rad.}^2)$.

(*e*) (i.) Calculate the area of a sphere 6 ft. in diameter, using each of the three above expressions in turn.

(ii.) Find, by measurement, the area of one face of a cube, and thence determine the area of the whole surface of the cube.

(iii.) Find the area of the surface of a rectangular block of wood not cubical. You should find the area of each face separately, and add the results together.

(iv.) Find the area of the outside surface of an oblong box of wood or cardboard without a cover.

24. Surface of a Cylinder.

(*a*) Cut out a piece of paper which will just cover the curved surface of a wooden cylinder having the curved surface at right angles to the ends, that is, a *right* cylinder.

The paper will be rectangular in shape ; and
Base of rectangle = circumference of cylinder,
Height of rectangle = height of cylinder.

The area of the paper is thus the same as that of the curved surface of the cylinder. Find this area.

This exercise proves that in order to find the *area of the curved surface of a cylinder* it is necessary to multiply the circumference of the base by the height of the cylinder. You also know that the *area of each circular end* is equal to the square of the radius multiplied by $3\frac{1}{7}$. Use this knowledge to work out the following sums:

(*b*) (i.) Find the area of the curved surface of a cylinder 10 in. high and 2 in. diameter.

(ii.) Find the area of the whole surface of a cylinder, with plane ends, 6 ft. high, and 2 ft. radius.

(iii.) How many square feet of paper would be required to cover the curved surface of a cylindrical pillar 20 ft. high and 2 ft. in diameter?

(*c*) (i.) Find, by measurement, the total surface area of a right triangular prism, that is, a solid having three rectangular sides and triangular ends.

(ii.) Find the total area of the outside surface of a box or drawer, neglecting the area of the edge. Find also the area of the inside surface.

(iii.) Find the area of the outside curved surface of a short piece of glass or brass tubing. Find also the area of the inside of the tubing.

(iv.) Measure the outside and inside diameters of a flat india-rubber ring, and use your measures to determine the total surface of the india-rubber, neglecting the edges.

25. Use of a Simple Balance.

(*a*) Uncover your balance, and notice the delicacy of its parts. Identify the different parts by reference to Fig. 40.

Raise the beam *AB* of the balance off the supports, by turning the handle *C* provided for that purpose.

Notice whether the index *F* swings equally on both sides of the middle of the scale *G*: if it does, the balance is ready for use; but if not, let down the beam and turn the small screw at

B; then try again. Repeat this adjustment until the swings to right and left are equal. Good balances ought to have adjusting screws at both ends of the beam.

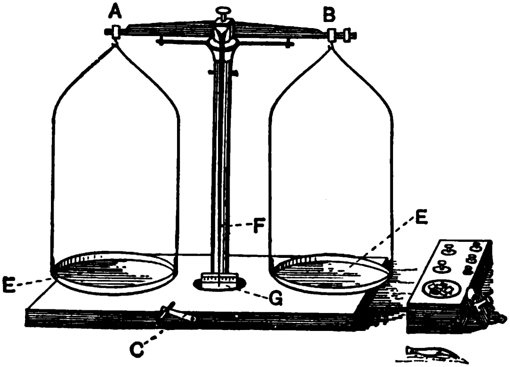


FIG. 40.—Students' balances and box of weights.

The mass of any body is the quantity of matter it contains. In our country the standard or unit of mass is the quantity of matter contained in a lump of platinum of a certain size which is kept at the Exchequer Chambers. This amount of matter is called the *imperial standard pound avoirdupois.* In France and a number of other countries the

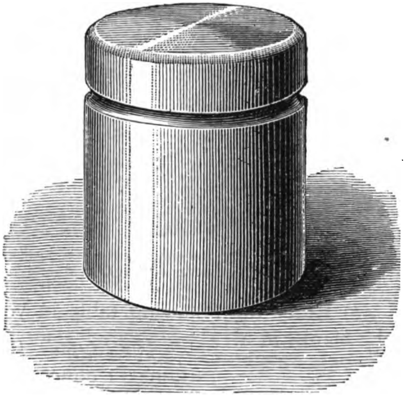


FIG. 41.—Exact size and shape of the British standard pound, made of platinum. From Aldous's *Elementary Course of Physics* (Macmillan).

standard is a certain mass kept at Paris, and called a *kilogram*. The system of masses founded upon it is used in all scientific work throughout the world.

The pieces of metal of different sizes used in weighing, and known as *weights*, are fractions or multiples of the standard mass. Examine the set of British weights supplied you, and write in your note-book the value of each weight.

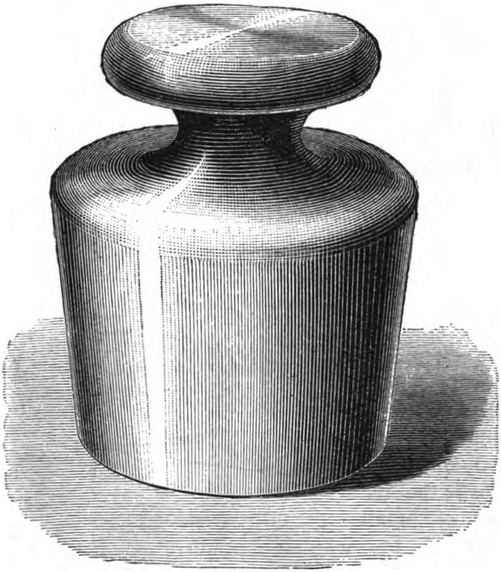


Fig. 42.—Exact size and shape of the standard kilogram, made of platinum. From Aldous's *Elementary Course of Physics* (Macmillan).

(*b*) Enter the following table in your note-book, and fill up the blanks in it.

Avoirdupois Weight.

...... drams = 1 ounce (1 oz).
...... oz. = 1 pound (1 lb.).
...... lbs. = 1 stone (1 st.).
...... lbs. = 1 quarter (1 qr.).
...... qrs. = 1 hundredweight (1 cwt).
...... cwts. = 1 ton.

(*c*) Put some shot in a pill-box or watch-glass and place the box in the *left-hand pan E* of the balance.

Examine your box of British weights, and, selecting a weight which you estimate to have about the same mass as the shot, take hold of it *with the forceps* and place it in the *right-hand pan* (the weights should always be placed in this pan).

Now slightly raise the beam to see whether the estimated weight nearly balances the shot. If the weight appears a little below what is wanted, pick up, with the forceps, the next greater weight in the box, and try it in the pan with the other. If the two together are too heavy, take out the smaller weight and put in the one below it, and so on, adding one weight after the other, *without missing any*, until you find the weight which just balances the shot.

Remember always to work down in the way described, from the largest to the smallest weight.

When you have completed the weighing, write down and add up the weights that are missing from their places, and check the figures as you put each weight back in its place.

(*d*) Find, by balancing against standard weights, the masses in English measure, of as many of the following coins as you can easily obtain :

Mass of a crown.......................................
" " a half-crown................................
" " five shillings................................
" " two shillings and a sixpence............
" " three pennies..............................
" " five halfpennies............................

(*e*) Examine the box of metric weights. The weight with 1 stamped upon it is known as 1 gram, and the numbers stamped upon the weights larger than it, denote the number of grams. The small flat weights are fractions of a gram.

You will remember that the metre is sub-divided into 10 decimetres, 100 centimetres, and 1000 millimetres.

The gram is divided in a similar manner into 10 decigrams, 100 centigrams, and 1000 milligrams. And just as 1000 metres is known as a *kilo*metre, so 1000 grams is called a *kilo*gram. With these facts and a little consideration you should be able to put the numbers in the blanks in a table like the following, which you should enter in your note-book :

...... milligrams (mgm.) = 1 centigram (cgm.)
...... cgm. = 1 decigram (dgm.)
...... dgm. = 1 gram (gm.)
...... gm. = 1 kilogram.

Write in your note-book a list of all the metric weights in your set.

(*f*) Using a pair of scales (not the delicate balance) place 1 kilogram in one pan, and put nails, shot, or sheet lead in the other pan until the two counterpoise. Then take out the 1 kilogram weight, and put British weights in instead until the things in the other pan are again counterpoised. In this way you find how many lbs. and ounces are roughly equal to 1 kilogram.

The comparison could be made direct by placing a kilogram weight in one pan and counterpoising it with British weights in the other. But by adopting the method described—known as *weighing* by *taring*—an accurate determination can be made even if the balance screws have not been adjusted as explained in Exercise 25 (*a*).

(*g*) Find by weighing, and calculating where necessary, the numbers required to fill the blanks in the following table, which should be entered in your note-book :

British Units of Mass to Metric.

1 grain = 0·065 gram.
1 oz. av. = ... „
1 lb. av. = ... kilogram.

Metric Units of Mass to British.

1 gram = ... grains.
1 dekagram = ... oz.
1 hektogram = ... ozs.
1 kilogram = ... lbs.

26. Determination of Areas by Weighing.

(*a*) Cut out a cardboard square *ABCD* with sides 8 cm. or 10 cm. long. From the same sheet of cardboard cut out a circular disc having a diameter equal in length to one of the sides of the square. Divide the large square into four smaller ones equal to one another by drawing lines connecting the middle points of opposite sides. The length of a side of a small square is then the same as the radius of the circle, and each small square, therefore, represents the square on the radius of the circle. Cut up the large square into the four small ones.

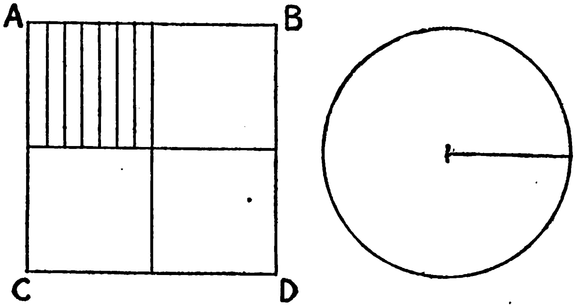


FIG. 43.—The area of a circle is equal to $3\frac{1}{7}$ times the square of the radius.

Evidently the area of the circle is not equal to that of the large square *ABCD*; that is, it is not equal to 4 times the square on the radius. To find exactly the relation between the area of the circle and the square on its radius, place the circle in one pan of the balance and see whether 1 of the small squares will counterpoise it. It will not. Try 2 squares, then 3. Still a little more is required. Divide the remaining square into seven or fourteen strips of equal width, and find how many of these strips have to be added to counterpoise the circle.

As the cardboard you use is presumably uniform in thickness, the equality shown by the balance also holds good for equality of area. Assuming this, you will find

area of circle $= 3\frac{1}{7}$ times the square on the radius.

(*b*) Repeat the exercise with a circle of different radius. In this case do not cut up the large square, but gradually cut away strips from one of the small squares drawn upon it, until the piece of cardboard just counterpoises the circle. Then find by measurement what fraction the part of the fourth square remaining is of the square on the radius.

(*c*) Upon the piece of cardboard or thin sheet brass of uniform thickness supplied you, draw a square with a side of a given number of centimetres, say 5. Cut out the square and find its mass in grams.

Since the square has a side of 5 cm., its area is

$$5 \times 5 = 25 \text{ sq. cm.}$$

Therefore the mass of 25 sq. cm. is a certain number of grams. Calculate the mass of one sq. cm. Repeat the experiment with pieces of cardboard or brass of known areas. Record your results thus :

AREA OF CARDBOARD OR BRASS IN SQ. CM.	MASS IN GRAMS.	MASS OF 1 SQ. CM.
1st piece, 2nd piece, 3rd piece, 4th piece,		
Total,		

Mean Mass of 1 sq. cm., - ______

(*d*) Weigh four irregular pieces of cardboard or sheet brass

A, *B*, *C*, *D*, of unknown areas, and knowing the mass of one sq. cm., calculate the areas of the various pieces.

Mass in Grams.	$\frac{\text{Mass in Grams}}{\text{Mass of 1 Sq. Cm.}}$ = Area in sq. cm.
Piece *A* =	= =
„ *B* =	= =
„ *C* =	= =
„ *D* =	= =

(*e*) Draw a circle of any convenient size upon the same kind of cardboard or sheet brass as was used in the preceding exercise. Cut out the circle and find its mass. As you already know the mass of 1 sq. cm., you can find the area of the circle in square centimetres. Measure the radius of the circle in centimetres, and calculate the square of the radius. Find how many times the value obtained will go into the area of the circle.

Mass of 1 Sq. Cm.	Mass of Circle.	Area of Circle.	Radius of Circle.	Radius Squared.	$\frac{\text{Area of Circle}}{\text{Radius Squared}}$.

(*f*) Draw upon thin cardboard two parallelograms between the same parallels, and having equal bases, as in Fig. 44. Cut out the parallelograms, and place one in each pan of the balance. Do they counterpoise one another? Find the mass of each.

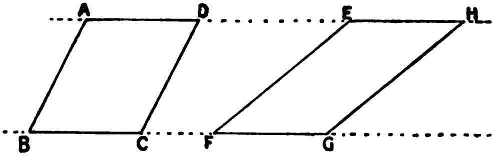


FIG. 44.—Parallelograms on equal bases and between the same parallels.

As in the preceding exercise, calculate the area of each parallelogram.

MASS IN GRAMS.	$\frac{\text{MASS IN GRAMS.}}{\text{MASS OF 1 SQ. CM.}}$	AREA IN SQ. CM.
Parallelogram *ABCD*		
„ *EFGH*		

(*g*) Draw upon card two triangles between the same parallels, and having equal bases, as in Fig. 45. Cut out the triangles,

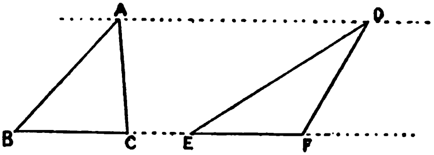


FIG. 45.—Triangles on equal bases and between the same parallels.

and place one in each pan of the balance. Do they counterpoise one another?

Find the mass of each triangle, and from the result determine the area as in the preceding exercise.

(*h*) Draw two right-angled triangles, and construct squares upon the sides of each as in Fig. 46. Cut out the squares. Place the largest square in one pan of the balance, and the two smaller squares together in the other. Do they counterpoise one another.

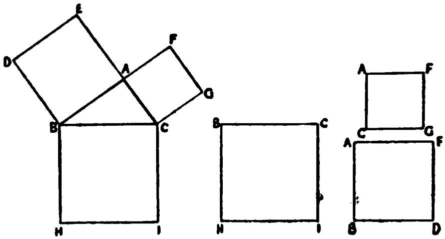


Fig. 46.—The square *BHIC* has the same area as the two squares *ABDE*, *ACGF* taken together.

Find the mass of the square constructed upon the hypothenuse of each triangle, and determine the area from the mass as before. Do the same with the two smaller squares taken together.

Now determine, by measurement, the area of each large square, and the total area of the two smaller squares belonging to each triangle. Record thus :

	Mass.		Area.	
	ABDE	*ACGF+BHIC*	*ABDE*	*ACGF+BHIC*
	Grams.	Grams.	Sq. Cm.	Sq. Cm.
1st triangle.				
2nd "				

Exercise 20 (*c*) in conjunction with this should have taught you an important fact with regard to the squares on the sides of a right-angled triangle. Express what you have learnt in your own words.

(*i*) Compare the area of England with that of Wales by gumming a map of England and Wales upon cardboard, cutting out the two countries, and weighing them. The relative masses obtained will show the relative areas.

CHAPTER III.

MEASUREMENT OF VOLUME AND CAPACITY.

27. Units of Volume.

(*a*) Examine your box of cubes.

How many faces or sides has each cube? How many edges has each cube? How many corners has each cube? What is the area of the face of one of the cubes, in British measure? What is the height of the cube, in British measure?

The record in your note-book should read somewhat as follows: "A cube has ... faces, ... edges, ... corners. The area of a face of the cube supplied is ... sq. in., and the height is ... in." You should remember always to write complete descriptions of your work, so that your note-book can be used for future reference, and can be understood by anyone reading it.

A cube, 1 inch long, 1 inch in breadth, and 1 inch in height, is a *cubic inch.* If it were 1 foot in each direction it would be a *cubic foot*; or if 1 centimetre in each direction, a *cubic centimetre.*

(*b*) Place four cubes to form a square slab upon your table: how many cubic inches do they make?

Now place a cube upon each of those lying on the table, so as to make two rows of 4 cub. in.: how many cubic inches are there altogether?

How many cubic inches would three rows contain, and how many seven rows?

You have already seen that the square of a number is the number multiplied by itself. The *cube* of a number is the number multiplied by itself twice. Thus, the cube of 6 is 6 × 6 × 6, that is, three sixes multiplied together, and the cube of 7 is 7 × 7 × 7. It is usual to write a small ^{3} on the top right-hand side of a number when the cube of a number has to be used, so that 8^3 means the cube of 8, or 8 × 8 × 8. In a similar manner 4 × 4 × 4 × 4 is written 4^4, and 12 × 12 × 12 × 12 × 12 is abbreviated to 12^5.

(*c*) Arrange eight one-inch cubes into a large cube. The length, breadth, and height of this cube are each 2 inches. The number of unit blocks in the large cube is, therefore, the cube of 2.

What numbers are the cubes of 2, 3, 4, and 5?

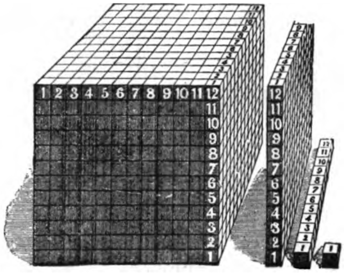


FIG. 47.—*A*, cubic inch; *B*, 12 cubic inches; *C*, 12 × 12 cubic inches; *D*, 12 × 12 × 12 cubic inches.

Fill up the blanks in the following table: Fig. 47 will help you to see how many cubic inches make one cubic foot.

Cubic or Solid Measure.

…… cub. in. make 1 cub. ft.
…… cub. ft. „ 1 cub yd.

(*d*) You know how many millimetres make one centimetre, how many centimetres make one decimetre, and how many decimetres make one metre. Use your knowledge to find the numbers to fill up the blanks in the following table:

…… cubic millimetres(c.mm.) = 1 cub. cm. (c.cm. or c.c.)
…… „ centimetres = 1 cub. decimetre.
…… „ decimetres = 1 cub. metre.

28. Relation between British and Metric Measures.

(*a*) Place eight one-inch cubes one on the top of the other so as to make a single pile. How many cubic inches are there in the pile?

Measure the length, breadth, and height in centimetres of the pile of cubes, and by multiplying the numbers together, determine the number of cubic centimetres there are in the pile. Find, by comparing the two results, how many cubic centimetres make one cubic inch.

Repeat the exercise with a smaller number of cubes.

CUBIC CMS.	CUBIC INS.	$\frac{\text{CUBIC CMS.}}{\text{CUBIC INS.}}$

(*b*) You know the proportion that the British measures of length bear to the metric measures. See whether you can calculate the numbers to fill up the blanks in the following table, as you have already done in the case of areas. This can be done at home :

British to Metric Measures.

1 cub. in. = ... cub. centimetres.
1 cub. ft. = ... cub. decimetres.
1 cub. yd. = ... cub. metre.

Metric to British Measures.

1 cub. centimetre = ... cub. in.
1 cub. decimetre = ... cub. ins.
1 cub. metre = ... cub. ft.
= ...... cub. yd.

29. Determination of Volumes of Simple Solids by Direct Measurement.

The preceding exercises have shown you that the volume of a rectangular solid can be determined by multiplying the area of the base by the height. Why this is so will be readily understood by an examination of Fig. 47, or better by means of blocks cut into slabs or cubes in the manner there shown.

(*a*) If the area of each small square of the base of a block like *A* in Fig. 48 is 1 sq. cm., and the slabs are 1 cm. thick, how many cubic centimetres are there in a single slab of the block? How many cubic centimetres are there in three of the slabs? How many cubic centimetres are there in the whole of the slabs?

Now examine a block like *B*, Fig. 48, and determine (1) the area of the base, (2) the number of cubic centimetres in a single slab, (3) the number of cubic centimetres in the whole of the slabs, that is, the volume of the triangular prism.

Proceed in the same way to determine the volumes of blocks like *C* and *D* in Fig. 48. The area of the base of *D* can be found by placing the block upon squared paper and drawing a line around it.

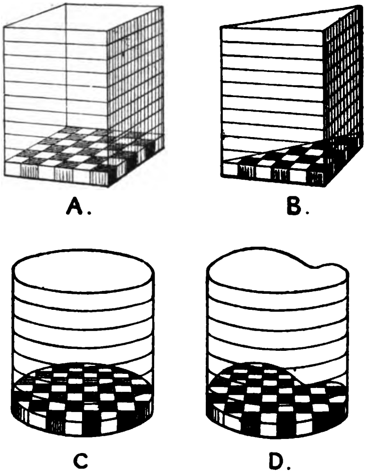


FIG. 48.—Determination of volumes of solids.

(*b*) Measure in centimetres the length, breadth, and height of a rectangular block, and by multiplying the three numbers together determine the number of cubic centimetres there are in the block.

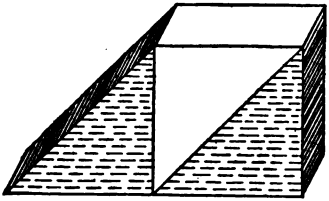


FIG. 49.—A parallelopiped can be converted into a cube.

Parallelopiped.

(*c*) Place together two triangular blocks or prisms to form a cube. Then place the shaded block against the unshaded one so as to form the slant shape shown in Fig. 49.

This shape is called a *parallelopiped.* Evidently the blocks have the same total volume whether they are placed to make a cube or a sloping parallelopiped.

Hence the cubic contents or *volume of a parallelopiped, like the volume of a cube, is equal to the area of the base multiplied by the perpendicular height.* (You will remember that a parallelogram has the same area as a rectangle of equal base and height.)

Triangular Prism.

Every triangular prism of the kind shown in Fig. 49, that is, having edges parallel to each other, and the faces at right angles to them, can be considered as half a rectangular block.

(*d*) Find the volume of the rectangular block formed by two triangular prisms, and from the result calculate the volume of one of the prisms.

(*e*) Select one of the prisms, and find the area of one of the triangular faces. Exercise 29(*a*) has taught you that to find the volume of the block, this area must be multiplied by the height. Do this, and compare your result with that obtained in the preceding exercise.

Cylinder.

(*f*) Find the area of one of the circular ends of your wooden cylinder, and also the height of the cylinder. From these numbers determine the volume of the cylinder.

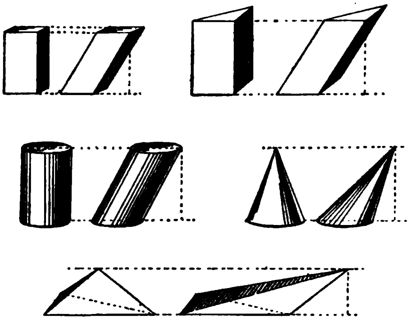


FIG. 50.—Pairs of solids having equal volumes, being on equal bases and of equal heights.

Upright and Sloping Solids.

It has been seen that the volume of a sloping parallelopiped is the same as that of a rectangular parallelopiped on an equal base and having the same height. In the case of some other solids also, the volume is the same whether they are upright or sloping. For instance, each of the solids of the pairs represented in Fig. 50 would have the same volume. This can be proved by measuring the volumes of as many of the pairs as are available, or, indirectly, by weighing. If two objects are made of the same kind of material, their volumes are evidently proportional to their masses, so that if one has twice the mass of another it has double the volume.

30. Comparison of Volumes by Weighing.

(*a*) Place a rectangular block in one pan of a balance, and a parallelopiped of the same height, base, and material in the other. You will find that the two will counterpoise, thus indicating that their volumes are equal.

Show the equality of the volumes of upright and sloping cylinders, pyramids, triangular blocks, etc., by comparing their masses.

(*b*) Select a cone having the same base and height as a cylinder of the same material. Weigh the cone and cylinder separately and compare the results. You will find by this means that the volume of the cone is one-third the volume of the cylinder.

(*c*) Compare, by weighing, the volume of a pyramid on a square base with the volume of a rectangular block having an equal base, and of the same height as the pyramid.

(*d*) Compare, by weighing, the volume of a sphere with that of a cylinder of the same height and diameter, and made of the same material. The sphere will be found to have two-thirds the volume of the cylinder.

By means of the preceding exercises you have determined practically the relations between the volumes of certain simple solids. It is a good mental exercise to develop these relationships from other considerations, and this may be done as follows:

31. Determination of Volumes by Geometrical Rules.

Pyramid.

A cube may be considered to be made up of six pyramids as indicated in Fig. 51. Suppose a cube of the same size be cut into six slabs as in Fig. 52. Then the volume of one of the pyramids is equal to the volume of one of the slabs, for each is one-sixth the volume of the cube. The volume of a slab is equal to the base multiplied by the height of the layer, which is one-third the height of a pyramid. Therefore, *the volume of a pyramid on a square base is equal to the base multiplied by one-third the height.* This rule applies to every pyramid. In other words, a pyramid could be flattened down to a block having the same base and one-third the height.

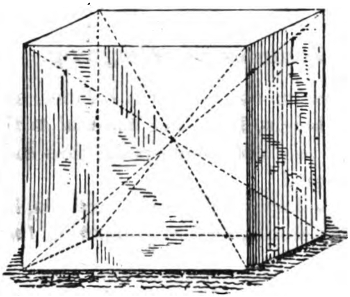


FIG. 51.—Cube formed by six pyramids.

(*a*) Find by means of this rule the volume of a pyramid on a square base.

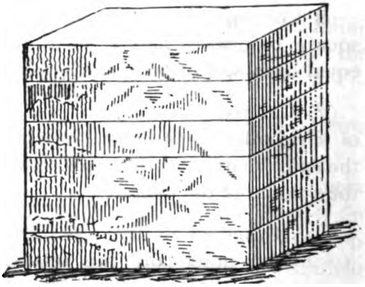


FIG. 52 —Cube formed by six slabs.

Cone.

A cone may be regarded as a pyramid with an infinite number of sides. The rule for obtaining its volume is, therefore, the same as for the pyramid. That is

$$\begin{aligned}\text{volume of cone} &= \tfrac{1}{3} \times \text{area of base} \times \text{height}\\ &= \tfrac{1}{3} \times \pi r^2 \times h\\ &= \tfrac{1}{3}\pi r^2 h.\end{aligned}$$

(*b*) Measure the height of the cone supplied and also the diameter, from which the radius, that is, one-half the diameter, is obtained. Using the numbers thus found, and the rule just given, calculate the volume of the cone.

Sphere.

The surface of a sphere can be divided into a large number of small triangles, each of which may be considered to form the base of a pyramid having a height equal to the radius of the sphere. Fig. 53 shows such a sphere with a few of the pyramids taken out. All the bases added together equal in area the surface of the sphere, and all the pyramids added together equal the volume of the sphere. Therefore the volume of the sphere can be found by multiplying the surface by one-third the height of the pyramids, that is, by one-third the radius. As has been previously shown :

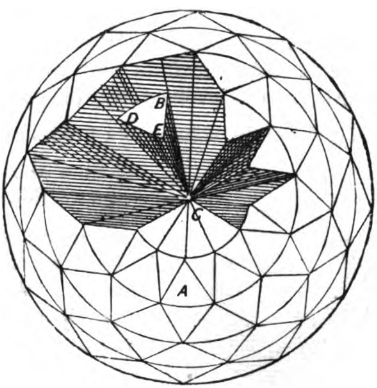


FIG. 53.—Sphere formed by numerous small pyramids. A few of the pyramids have been taken out.

$$\text{Surface of a sphere} = 4\pi \text{ rad.}^2 \text{ (p. 39).}$$

$$\begin{aligned}\text{But volume of a sphere} &= \text{surface} \times \tfrac{1}{3} \text{ rad.}\\ &= 4\pi \text{ rad.}^2 \times \frac{\text{rad.}}{3}\\ &= \tfrac{4}{3}\pi \text{ rad.}^3\end{aligned}$$

(*c*) Measure the diameter of a sphere and calculate its volume by means of the formula

$$\text{vol. of a sphere} = \tfrac{4}{3}\pi r^3,$$

r being the radius and π being $3\tfrac{1}{7}$.

Additional Exercises.

(*d*) Find the volume of a cylindrical box into which your sphere would exactly fit so as to be level with the top of the box. Calculate also the volume of the sphere, and determine what fraction this volume is of the volume of the box (Fig. 54).

(*e*) Calculate the volume of a cone having the same height as the cylinder and a base equal in area to one of the ends. What fraction is this of the volume of the cylinder?

(*f*) Find the volume of this book, as nearly as you can determine it. (As the front and back edges are not rectangular a little error is involved, but this may be neglected.)

(*g*) Find the volume of a brick in cubic inches.

(*h*) Find the volume of wood used in the construction of a certain drawer or box.

(*i*) Find the volume of a thick piece of glass—or brass—tubing about 5 cm. long. (Remember! the tube is not a solid cylinder, but less than such a cylinder by the volume of the space inside the tube. Fig. 54.)

(*j*) Find the volume of a new lead pencil. Then determine the area of the blacklead at the flat end of the pencil, and, assuming that it is the same size throughout, calculate the volume of the blacklead in the pencil. Subtract this volume from the total volume of the pencil, and so find the volume of the wood.

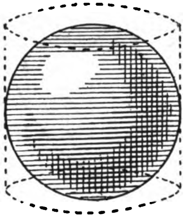


FIG. 54.—Sphere in its circumscribing cylinder.

32. Measures of Capacity.

(*a*) The half-pint measure supplied you is graduated into fluid ounces in apothecaries' measure. Test, by counting the number in the half-pint jar, how many fluid ounces there are in an Imperial Pint.

Apothecaries' Measure.

60 minims = 1 drachm.
8 drachms = 1 fluid ounce.
20 fluid ounces = 1 pint.
8 pints = 1 gallon.

(*b*) Select a medicine bottle having divisions for doses upon it. Find, by means of your jar graduated in fluid ounces, how much liquid is represented by the distance between two successive dose-marks. To do this pour the water from the bottle into the measuring jar.

(*c*) Cut out of cardboard a figure of the shape shown in Fig. 55 and of the size indicated by the numbers. Cut the cardboard half-way through at the dotted lines, and then bend it to form a cubical box which will hold 1 cubic decimetre, that is, 1 *litre*. Bind the edges together with glued tape, and varnish the box inside and out to make it watertight. Make a

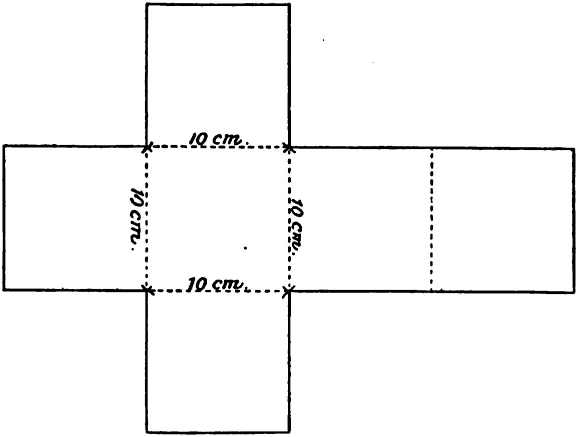


Fig. 55.—Construction of a box to hold 1 cubic decimetre, that is, 1 litre.

similar box having sides 1 in. in length, so that it will contain 1 cub. in. You will require these boxes again.

33. Relation between Cubic and Fluid Measures.

(*a*) Using your cubic inch box, find how many cubic inches of water are required to fill the half-pint measuring jar; and from this calculate how many cubic inches of water make a pint.

(*b*) Fill the cubic inch box with water, and pour the water into the graduated glass measure. In this way find the number of fluid ounces in 1 cub. in. Add, in succession, several more cubic inches of water. Observe the level in each case, and find the mean of the numbers obtained. Record thus:

			RISE.
1st level, - -	.... fluid ozs.	-	.... fluid ozs.
2nd „ - -	.... „	-	.... „
3rd „ - -	.... „	-	.... „
4th „ - -	.... „	-	.... „
		Mean	_____ fluid ozs.

Using the mean value thus obtained, find the number of cubic inches in 1 pint.

(*c*) Procure a narrow glass jar or bottle, and graduate it into cubic inches by filling your cubic inch box with water and emptying it into the jar. Mark the cubic inches with very narrow pieces of gummed paper.

> It is very difficult to make a box which *exactly* holds a cubic inch. You can judge the accuracy of your work by another and better way of graduating the bottle into cubic inches.

(*d*) The exact number of fluid ounces in one cubic inch is 1¾. Pour from the graduated measuring jar 1¾ fluid ounces of water into the bottle and make a mark on it level with the top of the water. How does this mark compare with the one you obtained with the cubic inch box? Repeat the operation until the bottle is nearly full of water, each time making the same comparison. This is a very good way of obtaining a vessel graduated into cubic inches.

34. Determination, in British Measure, of Volumes by Displacement.

(*a*) Pour water into the bottle, which you have divided into cubic inches, until the level rises exactly to a division. Then gently put sand or small stones into the bottle until the level of water reaches the next division. What is the total volume, in cubic inches, of the things you have dropped into the water?

(*b*) Fill the glass measure graduated in fluid ounces up to a mark about half-way up. Gently drop a stone or other irregular object into the water, and observe the rise of level. You know how many fluid ounces make 1 cub. in. The rise of level, therefore, gives you the means of determining the volume, in cubic inches, of the object used. Make this calculation.

The preceding exercises have shown you that it is possible to determine the number of cubic inches in an object which sinks in water, by noticing the number of fluid ounces displaced by the object, and remembering that 1¾ fluid ounces occupy 1 cubic inch. It is unfortunate that there is little connection between our cubic measure and fluid measure; but the metric system is far simpler, and you will find it much easier to determine volumes on this system than in British measures.

35. Vessels used in the Measurement of Volume and Capacity.

A number of vessels used in the measurement of volume are shown in Fig. 56. The three flasks marked 500 c.c., 250 c.c. and 100 c.c. have marks upon their narrow necks, and when filled up to these marks the numbers show how many cubic centimetres of liquid are in them. The tall jar has a mark at every 20 cubic centimetres up to 2000 c.c. The number of cubic centimetres of a liquid may thus be found by pouring the liquid into the jar and noticing the mark level with the surface.

The graduated tube is a *burette* used for measuring out exact quantities of liquid. At the bottom is a clip or tap for allowing the liquid to flow out of the burette. Suppose the burette were filled to the mark *o* and 35 c.c. of liquid were required from it. The tap would be gradually opened and when the liquid had fallen to the mark 35 c.c. it would be quickly closed. As burettes are usually used for measuring out liquid in this way, they are graduated from the top downwards as indcated.

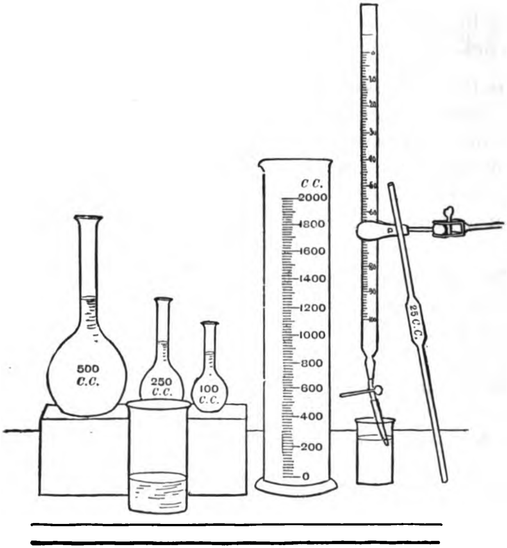


FIG. 56.—Vessels used in the measurement of volume and capacity.

The narrow tube by the side of the burette is a *pipette* by means of which small quantities of liquid may be conveyed from one vessel to another.

All these vessels will be used in the course of the following exercises.

36. Relation between British and Metric Measures of Capacity.

(*a*) Fill the cubic inch box with water. Pour the water into a metric graduated jar, and notice the number of cubic centimetres occupied by it.

(*b*) Fill the cubic decimetre box with water. Test, by measurement with a graduated jar, the number of cubic centimetres it contains. If accurately made, how many cubic centimetres should the box contain?

Use your graduated jars to determine:

(i.) The number of cub. cms. in 1 fluid ounce.

(ii.) The number of ounces and drachms in 100 cub. cms.

(iii.) The number of ounces and drachms in 1000 cub. cms.

(*c*) Fill a measure up to the half-pint mark with water. Pour the water into the metric graduated jar, and so find the number of centimetres in a half-pint. Repeat the operation, so as to determine the number of cubic centimetres in one pint.

(*d*) Pour 1000 c.c., that is, 1 litre, of water into a vessel. Then measure this amount of liquid in pints, so as to find the value of 1000 c.c. in British measure.

(*e*) The results you have obtained in the preceding exercises will enable you to work out the numbers to fill up the blanks in the following table, remembering that 1000 c.c. make one litre:

Equivalents of British and Metric Measures of Capacity.

1 pint = ... litre.
1 quart = ... litres.
1 gallon = ... litres.
1 litre = ... pints.

(*f*) Procure a large flask or bottle, and, using the graduated metric jar, pour 1000 c.c. of water into it. Make a mark at the level of the water. This bottle should be kept for use as a litre measure. Make a mark at the level of 500 c.c. so as to show the capacity of half a litre.

37. Use of Burette.

(*a*) Examine the burette supplied you, noticing especially its division into cubic centimetres. Fill it with water (it is an advantage to colour the water with a drop of red or black ink).

Notice that the surface of the liquid is not flat but curved downwards, being higher near the sides of the burette than at the centre.

In reading the level of a liquid having a surface of this kind, the lowest part of the curved surface should be observed, and the eye should be brought on a level with it, as shown in Fig. 57.

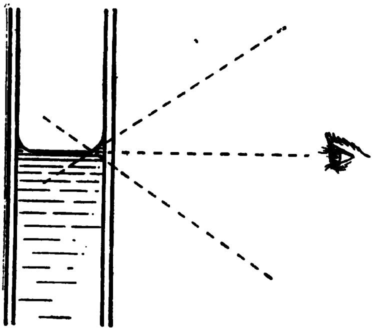


FIG. 57.—Showing that an inaccurate reading of the level of liquid in a burette is obtained, if the eye is not level with the surface of the liquid.

Find, by running in water from the burette, the number of cubic centimetres required to fill up the space between two successive divisions on the medicine bottle used in Exercise 32(*b*).

Record your results :

No. of c.c. in dose 1 = ... c.c.

„ „ doses 1 and 2 = ... c.c.

No. of c.c. in dose 2 = ... c.c.

„ „ doses 1 to 3 = ... c.c.

No. of c.c. in dose 3 = ... c.c.

(*b*) Using either water or mercury in your burette, graduate the tube supplied you into divisions 5 c.c. apart. To do this, stop one end with a cork, and make a mark on the tube with a three-cornered file about half an inch from the cork. Fill the tube with liquid up to the mark and then run in 5 c.c. more liquid from the burette, and make another mark on the tube.

Repeat this operation until you have obtained marks along the whole length of the tube.

> When mercury is used in a burette, the surface is seen to be curved in an opposite direction to that observed with water. In all cases of this kind, the highest point of the mercury surface should be taken, and the eye should be brought on a level with it when making an observation of the height.

(*c*) Obtain a small dry flask, place it under a burette, and deliver into it from the burette 20 c.c. of water.

(*d*) Fill a beaker and a narrow test-tube, or a flask with a narrow neck, up to a convenient mark with water. Run 1 c.c. of water from the burette into the beaker, and 1 c.c. into the narrow test-tube or flask. Measure the difference of level produced in each case. Again, add 1 c.c. to each, and measure the rise of level. The observations will show you the advantage of using narrow vessels when the rise of level produced by the addition of small volumes is to be measured.

38. Determination, in Metric Measure, of Volumes by Displacement.

(*a*) Fill the burette up to a convenient mark with water. Cut off about a metre of fine wire, make it into a small coil or bundle, and drop it into the burette. The water rises, being displaced by the wire. What is the volume of the wire?

> After dropping in the wire, tap the burette, or shake it somewhat so as to dislodge the air bubbles, and look to see that this has been done.

(*b*) Half fill with water the glass jar graduated into cubic centimetres; then gently put in a large marble, and see how many cubic centimetres are displaced by it. The number obtained is evidently equal to the number of cubic centimetres in the marble. Measure the diameter of the marble by means of calipers, and taking the mean of several measurements, calculate the volume. (Vol. of sphere $=\frac{4}{3}\pi r^3$.) Compare the result obtained by displacement of water with that obtained by measurement and calculation.

(*c*) Fill a test-tube up to a fixed mark with water from your burette, and write down the volume required. Empty the test-tube and dry it, then insert into it a number of small objects, as bits of lead, tin-tacks, etc., and again fill from the burette. How much liquid is now required to reach the same mark? What is the volume of the tin-tacks, etc.?

(*d*) Fill the graduated metric jar up to a known mark, and gently lower a stone into the water. Find the volume of the stone, by noticing the number of cubic centimetres displaced.

(*e*) Using a test-tube as in (*c*), find the volume of ten or twenty small equal steel balls, such as are used for bicycle bearings, and determine the volume of a single ball by dividing the observed volume by the number of balls used.

> To determine the volume of a solid lighter than water by the method of displacement utilised in the preceding exercises, a sinker sufficient to keep the object under water must be used.

(*f*) Obtain a small block of wood and a piece of sheet lead which will keep it under water. Tie a piece of thread to the lead, and then lower the lead into the graduated jar, but keep the loose end of the thread out of the water. Now pour water into the jar up to a certain mark. Observe the level of the water. Pull up the lead and tie it to the wood; then gently drop the two together into the water. The rise of level above that when the lead alone is immersed evidently shows the volume of the wood. Determine the volume of a cork in the same way.

(*g*) Compare, by the method of displacement of water, the volume of a right circular cylinder with that of a cone of the same base and height.

(*h*) Compare, by the method of displacement, the volume of a sphere with that of a cylinder of the same height and diameter, as in Exercise 31 (*d*).

(*i*) Find, by displacement of water, the volume of the piece of tubing used in Exercise 31 (*i*), or of a similar piece, and compare the result with that found by measurement and calculation.

CHAPTER IV.

MEASUREMENT OF MASS AND DENSITY.

39. Units of Mass in the Metric System.

In the metric system *the mass of pure water which will exactly fill a cubic centimetre at a temperature of* 4° C. is called a *gram*, and is the scientific unit of mass. The same prefixes are used to express fractions and multiples of a gram as have been used in the case of the metre and litre. The kilogram (see p. 42) is one thousand times greater than one gram, and is the unit in use for ordinary purposes.

The following exercise will show you approximately that the mass of 1 c.c. of water is one gram.

(*a*) Counterpoise a beaker or flask upon the left-hand pan of the balance. By means of the burette run 10 c.c. of water into the flask and find the mass of the water in grams. Find, in the same way, the mass of 20, 50, 100, and 200 c.c. of water. Record the results in parallel columns.

VOLUME OF WATER, C.C.	MASS OF WATER, GRAMS.

What is the mass of 1000 c.c. of water?

40. Densities of Solids.

(*a*) Obtain several cubic centimetre pieces of different substances, such as lead, iron, oak, pine, wax, cork, etc., and find the mass of each. You will thus find that the same volumes of different substances have different masses.

> *The mass of the unit volume of a substance is known as the density of that substance.* Taking the cubic centimetre as the unit volume, the mass in grams of each of the cubes used in the preceding exercise is the density.

(*b*) Determine, by measurement and calculation, the number of cubic centimetres in one of your wooden blocks. Then find the mass of the block employed in grams. You will be able from the two results to ascertain the mass of one cubic centimetre of the wood, that is, the density of the wood.

VOLUME OF BLOCK IN C.C.	MASS OF BLOCK IN GRAMS.	MASS OF 1 C.C.

(*c*) Find the masses of several of the simple solids of which the volumes have been determined in previous exercises. Calculate in each case the mass of one cubic centimetre, that is, the mass of unit volume, or the density.

VOLUME IN C.C.	MASS IN GRAMS.	MASS OF 1 C.C.

(*d*) Obtain cubes or cylinders of metal and wood and find their volumes by measurement. Weigh each solid measured and determine the mass of 1 c.c. of the substance.

(*e*) Calculate, or find by the method of displacement, the volume of a short length of glass or metal tubing. Weigh the tubing, and from the two results determine the density.

(*f*) Find the mass and the volume (by displacement of water) of a large stone marble, as in Exercise 38 (*b*), and thence determine the mass of one cubic centimetre of the marble, that is, the density of the marble.

(*g*) Make a file-mark on the neck of a flask or tall narrow medicine bottle, and fill the bottle with water up to this mark. Place half-a-dozen or so iron nails, the mass of which you have previously determined in grams, in the left-hand pan of the balance with the bottle, and counterpoise the whole. Now take the bottle out of the pan, place the nails inside it, and remove the water displaced by them. A convenient way to do this is by means of a *pipette* (Fig. 58). Put the bottle back in the pan, and find by adding weights the mass of the water removed.

Each gram represents one cubic centimetre of water. The number of grams added will therefore tell you the number of cubic centimetres displaced by the nails, that is, the total volume of the nails. Calculate the density of the nails.

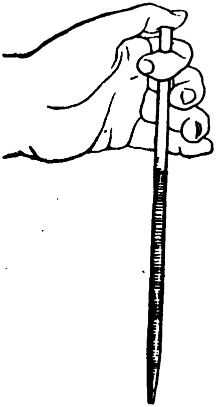


FIG. 58.—Use of a pipette for transferring small quantities of water.

(*h*) Employing the same methods find the mass of one cubic centimetre of each of the following substances: coal, marble, bronze (penny), silver (shilling).

41. Densities of Liquids.

(*a*) Counterpoise a small flask. Run about 30 c.c. of water into it from a burette, noticing exactly the volume used, as indicated by the difference of the two levels on the burette. Find the mass of this volume by weighing: thence calculate the mass of 1 c.c. Find in the same way the mass of 1 c.c. of other liquids.

(*b*) Counterpoise a flask having a narrow neck with a mark upon it. (You have already learnt that the mass in grams of the water which the flask would hold is approximately equal to the volume in cubic centimetres.) Fill the flask with various liquids in succession. Find in this way the mass of 1 c.c. of each of the liquids used.

VOLUME OF LIQUID IN C.C.	MASS OF LIQUID IN GRAMS.	MASS OF 1 C.C. OF LIQUID.

(*c*) For the determination of the density of a liquid, a tube of the form shown in Fig. 59 has several advantages over a flask

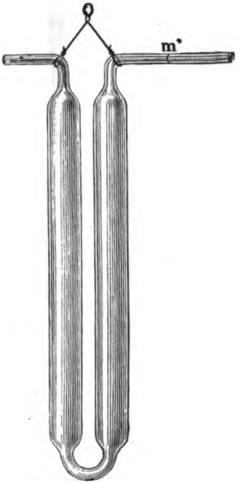


FIG. 59.—A Tube with fine bore near ends, for the determination of the densities of liquids.

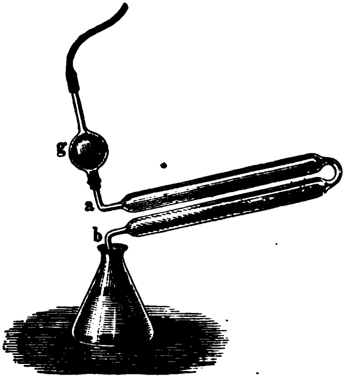


FIG. 60.—Method of filling a density tube having a fine bore.

having a mark upon the neck. The tube is filled with liquid by putting one of the open ends in a vessel containing the liquid, and attaching to the other end a pipette, by means of which suction can be applied (Fig. 60). When the tube is full, the pipette is taken off, and the tube is then ready to be suspended from the balance. If such a tube is available, the densities of some liquids should be determined with it.

42. Use of a Density Bottle.

(*a*) Take a narrow mouth stoppered bottle of about 4 oz. capacity, and file a vertical groove upon the stopper. Weigh the bottle. Fill it with water and insert the stopper so that the bottle is completely filled. Wipe away from the rim the water that has escaped through the file mark, and again weigh. What mass of water does the bottle hold? Hence what *volume* of any liquid?

(*b*) Again fill the bottle with methylated spirit, vinegar, turpentine, ink, or any other liquid, and find the mass of each liquid the bottle contains when full. As you know the *volume*, calculate in each case the *density*.

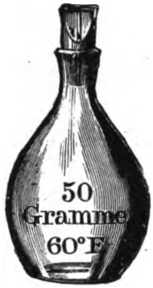


FIG. 61.—A bottle for determining relative density.

(*c*) Weigh some small pieces of lead and place them in the bottle. Fill the bottle with water and shake it in order to get rid of air bubbles; then insert the stopper, wipe the bottle, and weigh. How much less is the mass than the total mass of the lead and the bottle of water? This is obviously the mass of water forced out by the lead. What is its volume? and what then is the volume of the lead? Hence find the density of the lead.

(*d*) Find by the method used in the preceding experiment, the densities of such things as tin-tacks (these are really made of iron), bits of slate pencil, brass wire or brass nails.

In all the above experiments it has been assumed that the mass of 1 c.c. of water is exactly 1 gram. As this will be later found to be only the case at 4° C., the experiments have really only given the relative density with respect to water at the temperature of the experiment. The difference, however, will be very small.

CHAPTER V.

HYDROSTATIC METHODS OF DETERMINING DENSITIES.

43. Relative Densities of Liquids by Balancing Columns.

The ratio of the *mass* of a substance to the *mass* of an equal volume of water is termed the *relative density* of the substance. The term *specific gravity* is also applied to the same ratio, but it is convenient to use this expression only when the *weights* of unit volumes are compared.

(*a*) Cut off two pieces of glass tube of equal bore, each about 30 cm. long; connect the tubes with india-rubber tubing about 18 cm. long, and fix them upright upon a strip of wood, as in Fig. 62. Or bend a piece of glass tubing into the form of a U and mount it upon a stand, as shown in Fig. 63.

Pour mercury into one of the tubes until it reaches a horizontal line drawn a short distance above the bend (Fig. 63). Now introduce water into one of the tubes by means of a pipette, and notice that the mercury on which the water rests is pushed down; afterwards introduce sufficient water into the other tube to bring the mercury back to its original level. Measure the length of each column of water. Repeat the experiment with varying amounts of water.

(*b*) Nearly fill one of the tubes with methylated spirit, or some other liquid of which you already know the density, and balance it with water introduced into the other tube. Measure the lengths of the two columns.

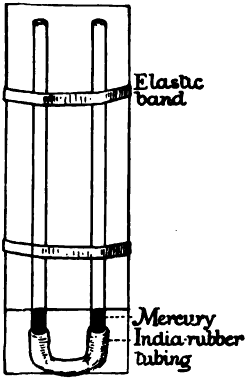


FIG. 62.—U-tube with india-rubber connections.

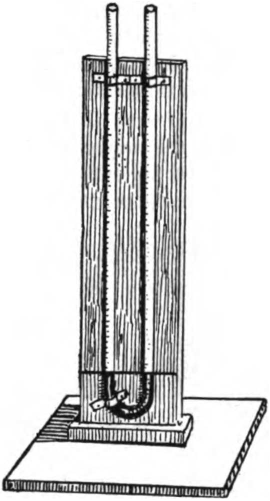


FIG. 63.—Glass U-tube on stand.

Compare the heights of the columns with the densities of the two liquids. What do you find?

From the result of the foregoing comparison, you will be able to understand that the densities of two liquids balanced in a U-tube are in inverse proportion to the lengths of the columns; in other words, the density of one liquid is less than the density of the other in proportion as its column is longer than the balancing column of the other. If water is used in one of the tubes, you can determine the relative density of the other liquid from the following proportion:

Density of other liquid : density of water
:: water column : column of other liquid.

Therefore relative density of other liquid

$$= \frac{\text{length of water column}}{\text{length of other liquid column}}.$$

(*c*) Find, by means of a **U**-tube, the relative densities of some of the liquids previously experimented with, and compare the results obtained by the different methods.

> The mercury in the bend of the **U**-tube is not essential when the liquids, the densities of which are being compared, do not mix with one another.

(*d*) Empty the **U**-tube. Pour water in until it fills about one-third the height of the tubes; then gently pour turpentine into one of the tubes until it nearly reaches the top. Let the liquids stand at rest for a few minutes; then make a mark upon the stand at the level where the two liquids join, as at AB in Fig. 64. Measure the lengths of the columns AC, BD in your **U**-tube, and thence determine the relative densities of water and turpentine.

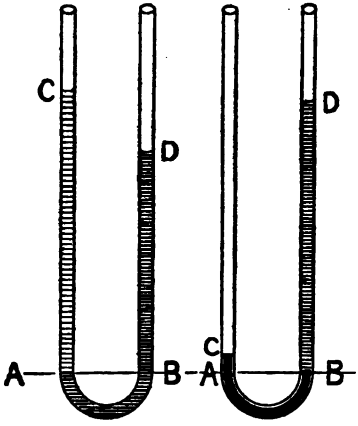


FIG. 64.—Determination of the densities of liquids which do not mix, by balancing columns in **U**-tubes.

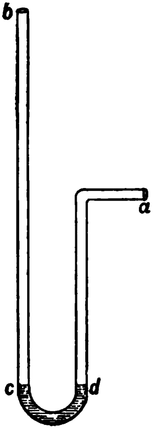


FIG. 65.—Glass tube for connecting with the gas supply to measure its pressure.

Compare in the same way the relative densities of mercury and water, and of oil and water.

(*e*) Slip a short length of india-rubber tubing over one end of the **U**-tube or over the end *a* of a tube bent like that in Fig. 65. Connect the rubber tubing with the gas supply. Pour a little water into the U-tube and notice the level. Turn on the gas. You will notice that the pressure of the gas forces the water down the tube connected with the supply, and up the open one. Measure the difference of level produced by this pressure. You thus find the length of a column of water which can be supported by the pressure of the gas. Repeat the exercise with mercury instead of water in the U-tube, and compare the lengths of the columns sustained by the pressure of gas.

The pressure of water from the main can be measured in the same way with mercury in a U-tube, but the U-tube should be a long one.

44. Hare's Apparatus for Determination of Relative Densities of Liquids which Mix.

Hare's apparatus is represented in Fig. 66. Two straight glass tubes are connected at the top by a three-way junction, upon the unconnected end of which a piece of india-rubber tubing is placed. The lower ends of the tubes dip into beakers containing the liquids the relative densities of which have to be determined. By applying suction to the free end of the rubber tube, the two liquids are drawn up the glass tubes, and the heights of the liquid columns *above the level of liquids in the beakers* will be inversely proportional to the relative densities of the liquids employed. The principle is thus precisely the same as that of the U-tube, but by using the form of apparatus here described, the relative densities of liquids which mix can be more conveniently found than by the ordinary U-tube in which the liquids have to be poured.

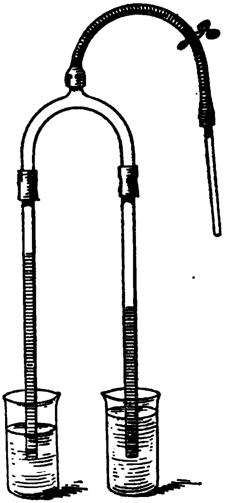


FIG. 66.—Hare's method of comparing the densities of liquids which mix.

(*a*) Use Hare's apparatus to measure the densities, compared with water, of as many of the following liquids as you can obtain: Mercury, turpentine, methylated spirit, saturated solution of copper sulphate, saturated solution of common salt, milk, vinegar.

(*b*) Fit up a Hare's apparatus with one tube larger in diameter than the other. Place the tubes in water and suck up the liquid as before. Is the level of liquid in both tubes the same? Substitute one of the liquids previously used for the water, and test whether the comparative lengths of columns of water and the other liquid are the same as when tubes of equal diameter were used.

> You will find that the height of the liquid columns is independent of the size of the tube *unless very narrow tubing is used.* This is true for all the experiments with the U-tube.

45. Siphon.

(*a*) Make a siphon by bending a glass tube in a fish-tail gas flame; make one limb about 6 inches long and the other at least a foot.

Fill the siphon with water, either by placing it in a bucket of water and covering each open end with one of your fingers before lifting it out; or, by sucking water through it as when using your pipette. Allow the siphon to empty itself; from which end does the water flow?

Fill the siphon again, dip the short limb in a beaker of water, and notice what occurs when you take your finger from the long arm.

Fill again and let the water flow into a tall narrow vessel. Keep the beaker full of water and notice when the flow of water stops.

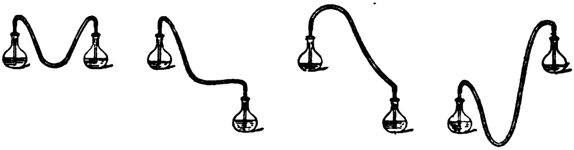


FIG. 67.—Flasks connected with tube to show the action of a siphon.

(*b*) Connect two short pieces of glass tubing with india-rubber tubing. Fill the tubes with water, and insert the ends below the surface of water in beakers or flasks about half full. Lift one of the flasks, and notice the flow of water which takes place. Show that no flow occurs when the level of liquid is the same in both vessels. Test whether a flow takes place when the bend of the india-rubber is below the lower of the two vessels (Fig. 67). Observe whether a siphon will act if there is a hole in it.

46. Principle of Archimedes.

(*a*) Suspend from a spring balance a metal ball or cube or cylinder, the volume of which is known or can be calculated from the dimensions. Or, instead, use any object, such as a glass stopper, the volume of which has previously been determined by displacement of water. When you know the volume of a body in cubic centimetres, you also know the mass of an equal volume of water in grams. Determine the weight of the object when suspended from the balance in air; then immerse the object in a glass of water and observe the loss of weight. To obtain a good result in this and other cases in which a substance is weighed in water, the air bubbles which cling to the object should be removed by means of a camel-hair brush.

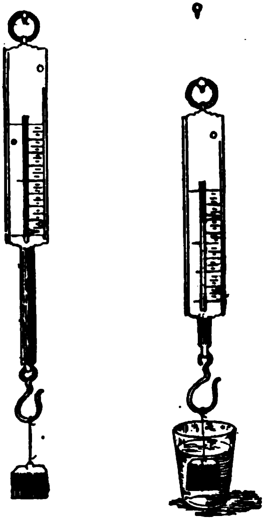


FIG. 68.—To show loss of weight experienced by immersing an object in water.

Weight in Air.	Weight in Water.	Loss of Weight.	Volume of Solid, c.c.	Weight of Equal Volume of Water.

A comparison of the third and fourth columns will show you that *when a body is submerged in a liquid it loses weight equal to the weight of the liquid displaced by it.* This important principle applies to all bodies, whether they are lighter or heavier than the liquid in which they are immersed.

(*b*) Repeat the preceding experiment, using another liquid, such as methylated spirit.

(*c*) Repeat with a pair of scales instead of a spring balance. To do this, suspend the object by means of a piece of thread from a hook above the left pan of a balance, and weigh it. Then place a glass of water under the pan so that the object is immersed, as in Fig. 69, and weigh again. The apparent loss of mass will, as before, be found to be equal to the mass of the volume of water displaced by the object used.

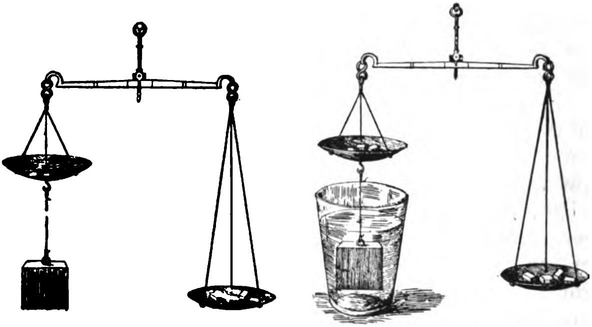


FIG. 69.—Scales arranged to determine the loss of weight experienced when an object is weighed in water.

(*d*) Fill a glass jar about two-thirds full of water, noticing the level of the water. Procure a cylindrical tin canister about half the diameter of the jar. Place the canister in the water, and gradually put shot into it until it just sinks in the water when the cover is on. Pour the water displaced by the canister into a beaker counterpoised upon a balance; then take out the canister, wipe it, and place it in the other pan of the balance. You will find that the mass of the canister and shot is practically the same as the mass of the water displaced.

(*e*) Repeat the preceding experiment, using another liquid, such as methylated spirit or turpentine instead of water.

A direct proof of the Principle of Archimedes can be obtained by means of a cylinder and a bucket or hollow cylinder into which it exactly fits (Fig. 70).

(*f*) Suspend such a cylinder from the left-hand pan of a balance; place the bucket in the same pan and counterpoise the two. Bring a beaker of water under the pan, so that the cylinder is immersed in it. A loss of weight is experienced. Gradually fill the bucket with water. When it is quite full, the balance will again be counterpoised, thus showing that the upthrust of the water on the cylinder is exactly equal to the weight of an amount of water having the same volume as the cylinder.

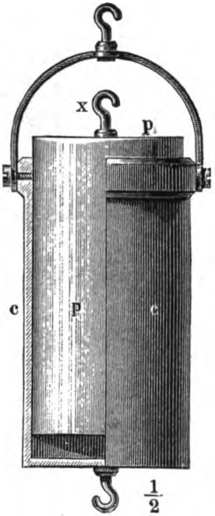


FIG. 70. — Cylinder and bucket for demonstrating the Principle of Archimedes.

Instead of a bucket and cylinder experiment, the following instructive modification of it may be performed.

(*g*) Suspend an object from the left-hand scale pan of a balance arranged for relative densities. Place in the pan a small measuring glass graduated in cubic centimetres. Counterpoise the suspended object and the measuring glass together. Now pour water into a graduated jar until the jar is about two-thirds full. Notice the level. Bring the jar under the short scale pan so that the object is immersed, as in Fig. 71. Notice the amount of water displaced. Put water gradually into the measuring glass by means of a pipette. When the balance

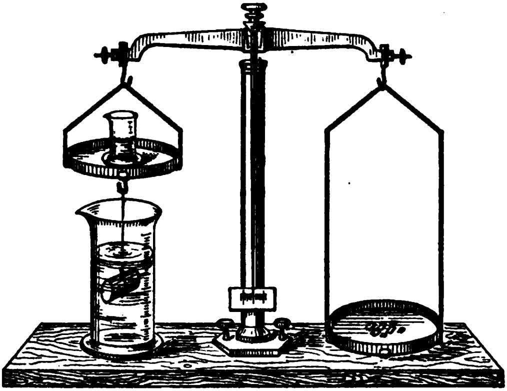


FIG. 71.—Experiment to prove that the loss of weight in water is equal to the weight of water displaced.

swings evenly, notice the amount of water which has been put in ; it will be found to be the same as the amount of water displaced.

47. Relative Densities by the Principle of Archimedes.

(*a*) Find the mass of a glass stopper or a marble, and also the mass of an equal volume of water. This may be determined as before or as follows : Suspend the stopper, by means of a piece of thread, from the hook above the left-hand pan of the balance, and weigh it. Then place a glass of water upon a platform or stage so that the stopper is immersed in it, as in Fig. 72, and weigh again. Record as below :

Experiment with Water and Glass Stopper.

Volume of glass stopper, - - ...... c.c.
Mass of equal volume of water, ... gm. (1)
Mass of stopper, - - - - ...... gm.
Apparent mass when immersed in water, ... gm.
Loss in water, - - - - ...... gm. (2)

(*b*) Determine the apparent loss of mass experienced by the glass stopper already used when weighed in succession in turpentine, methylated spirit, olive oil, and petroleum, instead of water.

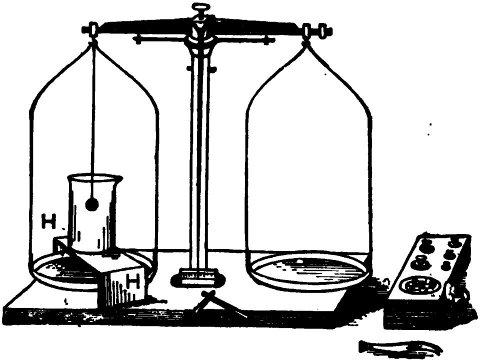


FIG. 72.—Hydrostatic method of determining relative density. *H*, *H* platform of thin wood or metal.

The apparent loss of mass in each case is equal to the mass of a portion of liquid of the same volume as the stopper. The numbers obtained therefore represent the masses of equal volumes of water, turpentine, methylated spirit, olive oil, and petroleum.

Record your observations as below:

	WEIGHING IN AIR.	WEIGHING IN LIQUID.	LOSS.
Water, - - -			
Turpentine, - -			
Methylated Spirit,			
Olive oil, - -			
Petroleum, - - -			

The last column gives the masses of equal volumes of the liquids used. Use these numbers to determine the relative densities of the liquids.

(*c*) Find by the method used in the preceding exercise, the relative densities of various substances, *e.g.* sulphur, copper, zinc, lead, glass, coal, and flint.

> To find by the Archimedes method the relative density of such a substance as wood, which floats in water, a sinker should be used, as follows;

(*d*) Weigh the small block of wood supplied, and obtain a piece of lead sufficient to sink it. Tie a piece of thread to the lead and suspend the lead in water as in the preceding experiments. Counterpoise with shot while the lead is under water and the wood is suspended above it in air. Now tie the block of wood to the lead; lower the two together into the water, and find what weights have to be put in the short pan in order to make the balance swing evenly. This is evidently the apparent loss of mass.

> This loss gives the mass of a volume of water equal to the volume of wood, and in connection with the mass of the wood in air enables the relative density of the wood to be determined. By suspending the lead in water both before and after the wood is tied to it, the result is not affected by the sinker.

(*e*) Repeat the preceding exercise with other solids which float in water, for example, cork, wax.

> It is not essential to use water in determining the relative densities of solids by the Principle of Archimedes. Wood sinks in spirit, and the relative density of it can be determined by weighing it in spirit without a sinker, provided you know either the mass of one c.c. of spirit, or the number of cubic centimetres of spirit which have a mass of one gram.

(*f*) The mass of one c.c. of spirit has already been determined by several methods. From one of your results calculate the number of cubic centimetres occupied by 50 grams of spirit, and the number occupied by one gram of spirit.

(*g*) Weigh a small block of wood in air. Suspend the block from the specific gravity balance, so that it is immersed in spirit. Determine its apparent loss of mass.

$$\text{Vol. of block in c.c.} = \frac{\text{apparent loss of mass}}{\text{mass of 1 c.c. of spirit}}.$$

Having obtained the volume of the block in this indirect way, the mass of an equal volume of water is known, and the mass of the block having previously been determined, the relative density of the wood can be calculated in the usual manner.

(*h*) Determine the mass of 50 cubic centimetres of a saturated solution of copper sulphate and thence calculate the mass of one c.c. Then determine the relative density of a copper sulphate crystal by weighing the crystal in air and in the saturated solution, as in Exercise 47 (*g*).

(*i*) A more convenient method is to use a liquid, such as petroleum, which does not dissolve the salt, the density of which is required. Using petroleum, determine the relative density of a crystal of washing soda.

48. Flotation.

(*a*) Obtain a rectangular rod of wood 1 sq. cm. in section and about 15 cm. long. Bore a hole in one end, fill it with lead, and smooth over the end with wax.

Weigh the rod.

Select a narrow glass jar graduated into cubic centimetres, or a burette; fill it with water up to a certain mark, and put the rod into it. Notice the length of rod immersed and how many cubic centimetres or grams of water are displaced by the rod (Fig. 73). Repeat the experiment with turpentine, brine, and methylated spirit, in

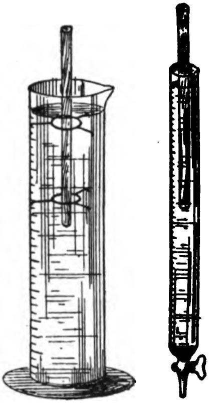


FIG. 73.—Method of determining the amount of liquid displaced by a floating rod.

each case running off the liquid displaced and weighing it. Record your observations as below:

Mass of Rod, gm.	Mass of Liquid Displaced, gm.

A comparison of the results thus obtained will show that the mass of the whole rod is equal to the mass of the liquid displaced by the portion of the rod immersed.

(*b*) Weigh one of your wooden cubes. Slightly oil the cube and place it in a glass of water. Mark upon the cube with a pencil where the surface of the water touches it. Then take out the cube and determine the volume of the immersed portion in cubic centimetres. You will then know the volume, and therefore the mass, of the water displaced.

Mass of Block, gm.	Volume of Immersed Portion of Block, c.c.	Mass of Water equal to the Immersed Portion of Block, gm.

In this case, again, it will be found that the mass of the whole block is equal to the mass of the volume of water displaced by the portion immersed.

(*c*) Scratch a line across the curved surface of a tin canister, at a short distance from the top. Float the canister in water without the cover, and drop shot into it until the water reaches the mark which you have made upon the curved surface. Show, as before, that the mass of the volume of water displaced is equal to the mass of the whole box and the shot. Repeat the experiment by putting more shot in the canister, and so making it sink deeper in the water.

(*d*) Fill the metric graduated jar with water up to a certain mark, and then float a narrow test-tube or glass tube closed at one end in it. Put sufficient shot into the test-tube to make it float upright; and, when it does so, notice the volume of water above the initial level of the water in the jar. The mass of this water in grams will be equal to the number of cubic centimetres above the initial level. Take out the test-tube and weigh it; the mass will be found equal to the mass of water displaced. Repeat the experiment with the test-tube floating at different depths.

VOLUME OF WATER DISPLACED, C.C.	MASS OF WATER DISPLACED, GM.	MASS OF TEST-TUBE AND SHOT.

49. Hydrometers and their Use.

(*a*) Procure a narrow test-tube and cut a strip of paper of nearly the same length. Mark the paper with lines at equal distances (about 5 mm.) apart and numbered from the bottom to the top, and gum it inside the test-tube so that the divisions can be seen through the glass. Put enough shot or mercury into the test-tube to make it float upright (Fig. 74). Cork up the tube, and observe the depth at which it floats in water, turpentine, and milk.

Evidently an instrument of this kind enables the density of a liquid to be determined by observing the level at which it floats when placed in the liquid. An instrument by which the density of a liquid is determined in this way is known as a *hydrometer.*

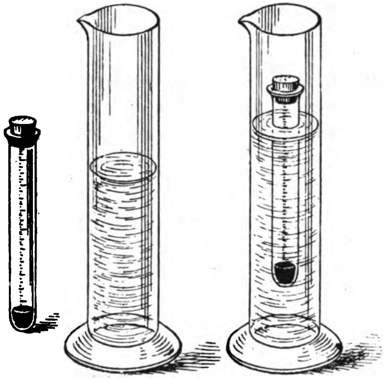


FIG. 74.—A simple hydrometer and its use.

(*b*) Examine the hydrometer supplied. Float it in a jar of water, and notice the division level with the surface (Fig. 75). Float the instrument in a liquid such as spirit or turpentine, the density of which is known, and see if this density is indicated by the hydrometer.[1] Determine by means of the hydrometer the density of milk, vinegar, and any other liquids available.

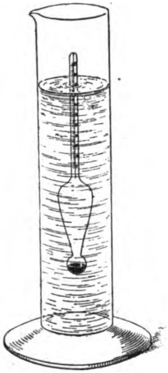


FIG. 75.—A hydrometer of variable immersion.

The hydrometers hitherto used are known as *hydrometers of variable immersion.* In another kind of hydrometer, known as *Nicholson's hydrometer*, the instrument is always immersed to a fixed mark upon it, and densities are determined by finding the masses necessary to produce this amount of immersion in different cases. The densities of solids as well as of liquids can be determined with this instrument.

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[1] If a "Twaddell" hydrometer is used, the rule for determining the density from the reading is, "Multiply the hydrometer reading by 5 and add 1·000." The reason for this is explained in Stewart and Gee's *Practical Physics*, vol. I.

(*c*) Place a Nicholson's hydrometer in a tall jar of water Load the top pan *A* until the mark on the stem of the hydrometer is level with the surface of the water. Record the load; then remove it and put in its place a pebble of less mass, or some other substance of which the density is required. Find the weights which have to be added to depress the hydrometer to the mark on stem. Then you have—

Mass of substance (a) + added mass (b) = mass required to sink hydrometer to mark (c).

Therefore mass of substance $a = c - b$.

Now put the substance in the lower pan *B*, and add weights until the mark is again reached. The loss in water is the difference between the weights now added and those required to depress to the mark when the substance was in the top pan. You have thus the mass of the substance and the apparent loss in water, and can therefore determine the relative density of the substance without a balance.

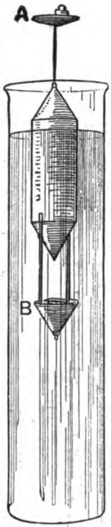

FIG. 76.—A hydrometer of constant immersion.

Determine the density of sulphur and lead by means of Nicholson's hydrometer.

(*d*) Weigh the Nicholson's hydrometer, and then place it in a jar of water. Add weights until the mark on the stem is level with the water. Then—

Mass of hydrometer + mass added = mass of a certain volume of water displaced.

Repeat the experiment with one or two other liquids. The same amount of liquid is displaced in each case, so you obtain the masses of equal volumes of liquids compared with that of an equal volume of water. Determine from your observations the relative densities of the liquids.

CHAPTER VI.

PRESSURES OF LIQUIDS AND GASES.

50. Relation between Pressure of Liquid and Depth.

(*a*) Bend a piece of glass tubing, as shown in Fig. 77, the long arm having a length of about 40 cm. Mount the tube firmly upon a strip of wood having half centimetre divisions marked upon it, or upon a metric scale. Pour enough mercury into the tube to just fill the bend.

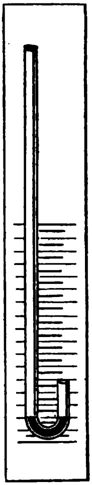

FIG. 77.—Arrangement for determining the pressure of water at different depths.

Notice that the level of the mercury is the same in the two arms of the tube.

Lower the frame into a tall jar of water so that the open end of the short arm of the tubes is 10 cm. below the surface of the water.

Notice the difference of level of the mercury in the two arms, and record it in your note-book.

Lower the tubes an additional 10 cm. and observe the effect. Then lower the frame as far as it will go, and again note the difference between the height of the mercury in the two arms. Record your observations in parallel columns thus:

DEPTH OF MERCURY SURFACE BELOW WATER.	DIFFERENCE OF LEVEL OF MERCURY PRODUCED BY WATER PRESSURE.

(*b*) Repeat the preceding exercise, using turpentine, or a strong solution of salt, or any other convenient transparent liquid instead of water.

By these exercises you will find that (1) the pressure increases from the surface of the liquid downwards, and is directly proportional to the depth; (2) the pressure at a given depth increases with the density of the liquid used.

To show that the pressure in any particular liquid depends only upon the depth, mount two tubes upon a frame, as shown in Fig 78—one with the lower opening turned upwards, and another with the opening pointing sideways.

(*c*) Pour the same quantity of mercury into the tubes. Arrange the tubes so that the lower openings are all at the same level. Lower the frame into water a few centimetres at a time, as in Exercise 50(*a*), and observe the

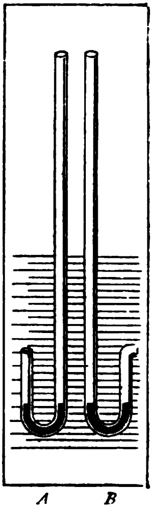

FIG. 78.—Arrangement for showing that pressure in any particular liquid depends only on depth.

difference of level of mercury produced in each tube. Record in columns as below:

DEPTH OF END BELOW SURFACE OF WATER.	RISE OF MERCURY IN *A*.	RISE OF MERCURY IN *B*.

(*d*) Immerse the frame of tubes to any level which causes the mercury to rise by a clearly-marked amount. Turn the frame to face various directions, keeping it at the same depth, and notice that the pressure as indicated by the rise of mercury is the same in all directions at any particular depth. Test this for several depths.

51. Relation between Pressure of Liquid and Area.

The preceding exercises have shown that the force or pressure upon a given area below the surface of a liquid increases with the depth. It will easily be understood that the larger the area the greater will be the total pressure upon it at any particular depth, though the pressure upon unit area is constant.

(*a*) Obtain two tubes of brass, glass, wood, or varnished cardboard, each about 15 cm. long, but one being about twice the diameter of the other. Close one end of each tube by means of a disc of the same diameter fixed upon it, or by a cork. Determine the area of the bottom of each tube, and find the proportion which one bears to the other.

Make a mark upon each tube at the same distance—say 10 cm.—from the closed end.

Float the tubes in a jar of water, and add sufficient shot to each to immerse them to the marks already made. The lengths immersed are then the same.

Take out each tube, wipe it dry, and weigh it with the shot it contains. Find the proportion of one weight to the other. Record thus:

$$\frac{\text{Area of end of large tube}}{\text{Area of end of small tube}} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots;$$

$$\frac{\text{Weight of large tube and shot}}{\text{Weight of small tube and shot}} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots.$$

As the bottom of each tube was at the same distance below the surface of the water, the pressure due to depth was the same. This pressure or force has been found to act in all directions, and it urges the tubes upwards.

The foregoing results thus show that the *total pressure* varies in proportion to the areas of the surfaces below any given depth. The student should therefore bear in mind that though the *pressure* at any given depth is constant the *total pressure* is dependent upon area.

52. Pressure of Liquid independent of the Form and Volume of Containing Vessel.

(*a*) Bend a short length of fairly wide glass tubing into a

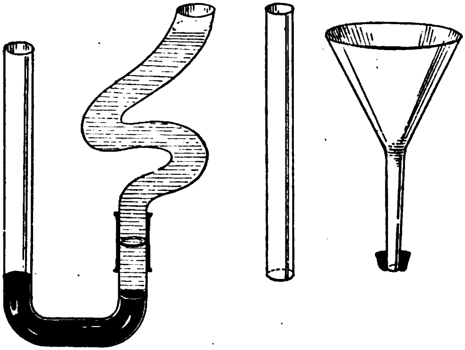

FIG. 79.—Experiment to show the pressure produced by liquids in vessels of different shapes.

U-form, as shown in Fig. 79. Fit a short piece of india-rubber tubing over one of the arms. Pour sufficient mercury into the tube to cover the bend. Connect a straight piece of glass tubing with the india-rubber, and introduce water into it. Measure the difference of height of the mercury in the two arms produced by the pressure of the column of water. Substitute a funnel, or a curved tube for the straight one, and fill it to the same height with water. Measure again the difference of height of the mercury in the two arms of the **U**-tube.

> Do the measures indicate that the pressure upon the mercury depends upon the *quantity* of liquid above it, or merely upon the *head* or height of liquid?

Test your conclusion by using tubes and vessels of various forms in succession as before, but with a different *head* of water, taking care, however, that the head is the same in each case.

53. Upward Pressure of Liquid.

(*a*) Procure a piece of wide glass tubing, or a straight lamp glass, having one end flat. If stiff leather is available, cut out a disc of slightly larger diameter than that of the glass, and pass a knotted thread through its centre. If a leathern disc cannot be obtained, make a disc of wood or stiff cardboard.

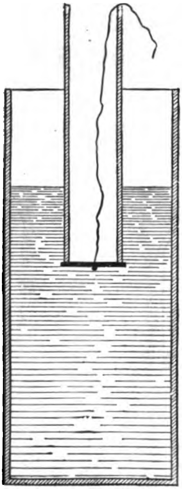

Fig. 80. — Upward pressure of liquid.

Hold the disc tightly upon the flat end of the glass by means of the thread, and while doing so lower the glass into a jar of water (Fig. 80).

> When the end of the glass with the disc upon it is a few inches below the surface of the water, the thread can be released and the disc will be found to remain in its position.
>
> Explain why the disc does not fall off the end of the glass.

Pour water very gently into the inside of the glass, and notice the height (inside and out) at that moment when sufficient water has been introduced to make the disc drop.

Repeat the experiment with the glass immersed by different amounts.

(*b*) Fit tightly a piece of thin india-rubber over one end of the glass cylinder used in the preceding exercise. Pour water into the cylinder, and observe the form taken by the india-rubber. Lower the cylinder into a jar of water, and notice how the form of the india-rubber is affected. When the india-rubber becomes flat, is there any difference in the level of the water inside and out?

Continue to lower the cylinder, and observe and explain the change of form it undergoes.

54. Pressure of the Atmosphere.

(*a*) Select a thistle funnel or a tin funnel of the form shown in Fig. 81. Tie a piece of thin sheet india-rubber, such as that used in toy air-balls, over the top. While the rubber remains flat the pressure is evidently the same on both sides of it.

Blow into the funnel. Make a sketch of the form which the india-rubber now takes. Explain why the india-rubber is forced out.

Suck the air out of the funnel. Sketch the form now assumed by the india-rubber. What is pressing upon the outside and forcing the india-rubber into the funnel?

Suck the air out of the funnel and place your thumb over the open end so as to prevent air from entering. Turn the funnel in various directions and see whether you can detect any difference in the amount of bulging of the india-rubber. If not, you may conclude that the pressure upon the outside is the same in all directions. What is it that is exerting the same pressure in all directions?

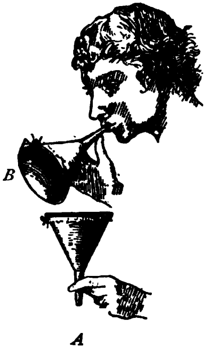

FIG. 81.—*A*, funnel with flat membrane over the top: *B*, funnel with membrane made concave by the pressure of the atmosphere.

(*b*) Fill a gas collecting jar, or a tumbler having a flat rim, with water; cover it with stout paper and invert (Fig. 82). Why does not the water drop out?

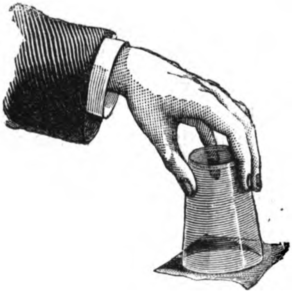

FIG. 82.—The paper remains on the glass on account of atmospheric pressure.

(*c*) Place in water one tube of the Hare's apparatus for determining density, and the other tube in mercury. Suck out the air. The liquids rise. Why? Notice the difference in the level of the mercury and the water. Explain the cause of this difference. Let one of the tubes of the Hare's apparatus be much wider than the other. Place the ends of the tubes in mercury, and suck out the air. Is there any difference in the height of the mercury in the large and small tubes?

55. The Siphon or U-Tube Barometer.

(*a*) Procure a board about 38 in. long, 4 in. wide, and ½ in. thick. At a distance of 4 in. from one end draw a line across the board, with Indian ink, and then draw another line exactly 28 in. above it. Cut a strip 4 in. long from squared paper divided into tenths of inches, and paste it upon the board so that the lowest edge is exactly over the upper line previously drawn. Number the inches upon this strip in the manner shown in the accompanying figure (Fig. 83).

Select a stout glass tube about 32 in. long, and closed at one end. Slip a piece of india-rubber tubing about 4 in. long upon the other end and fix it there with string or with copper wire. Fix the other end of the india-rubber upon a similar glass tube 5 or 6 in. long, open at both ends. Procure some dry mercury, and clean it by squeezing it through a clean duster, or through a blotting-paper filter. Rest the tube with its closed end downwards and pour the mercury into it until the liquid reaches the short glass tube. Shake out all the air-bubbles entrapped by the mercury, and fix the tube upon the long strip of wood, in the manner shown in Fig. 83. Observe and describe what happens. What is there between the top of the mercury and the closed end of the tube?

The instrument you have constructed is similar to a **U**-tube; can you say what is balancing the column of mercury?

(*b*) The preceding experiment will have shown you that air pressing upon the surface of the mercury in the short open arm of the **U**-tube will balance a long column of mercury in the closed arm.

Slip a piece of india-rubber tubing upon the open end and notice what happens when you blow sharply into it. Suck air out of the tube, and observe the result. These experiments show you the effect of increasing and decreasing the pressure upon the free surface of the mercury.

How will the height of the mercury column be affected (1) if the pressure of the air decreases, (2) if the pressure of the air increases?

Instruments like the one you have constructed, for measuring the pressure exerted by air, are called *Barometers.*

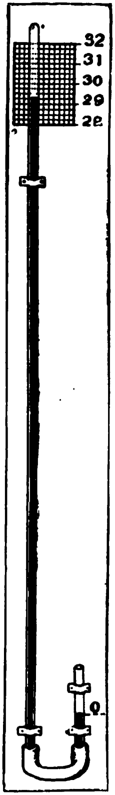

FIG. 83.—A barometer. The pressure of the atmosphere acting upon the open end of the short tube supports a column of mercury about 30 inches long in the closed tube.

56. The Cistern Barometer.

The following is another form of barometer :

(*a*) Procure a thick glass tube about 36 inches long and closed at one end. Fill the tube with mercury ; place your forefinger over the open end ; invert the tube ; place the open end in a cup of mercury and take away your forefinger (Fig. 84). Measure the distances from the surface *A* of the mercury in the basin to the top *B* of the mercury column.

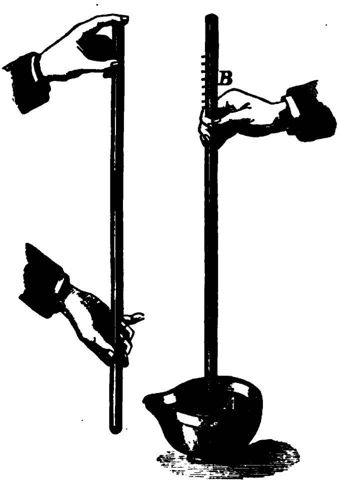

FIG. 84.—Construction of a barometer.

(*b*) Slant the barometer and measure the vertical height of the mercury by means of a plumb-line. Compare it with your previous reading.

A column of mercury will be supported in the tube by the pressure of the atmosphere. The distance between the top of the column and the surface of the mercury in the cup will be about 30 inches, or 76 cm., when the tube is vertical (Fig. 85, *b*). If the tube is inclined so that the closed end of it is less than this height above the mercury in the cup (Fig. 85, *c*) the mercury fills it completely ; and if the tube is less than 30 inches long, it is always filled by the mercury whether it is inclined or not (Fig. 85, *a*). On an average the atmosphere at sea-level will balance a column of mercury 30 inches in length. No matter if the closed tube is 30 feet long, the top of the mercury column will only be about 30 inches above the level of the mercury in the cistern.

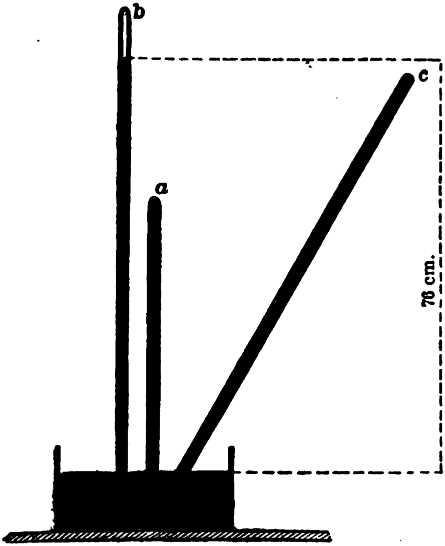

FIG. 85.—The atmosphere at sea level will support a column of mercury up to 30 inches, or 76 cm. in length.

57. Pressure of Vapours on Barometric Column.

(*a*) Bend one end of a pipette or a piece of narrow glass tubing about six inches long, and cut it off near the bend, so as to make a short hook, or procure a pipette bent at the lower end (Fig. 86). Place the hook underneath the bottom of the barometer, and blow into the other end, so as to force a bubble of air up the mercury in the tube. Observe and record the effect of the introduction of air upon the height of the mercury column.

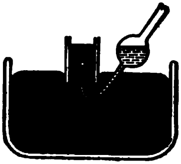

FIG. 86.—How to introduce a liquid into a mercury barometer.

(*b*) Introduce a little water into the barometer in the same way, and observe the effect. Is the effect due to the weight of the water added? If not, to what is it due? Try the effect of liquids other than water.

58. The Reading and Plotting of Barometric Heights.

(*a*) Many newspapers give charts showing the variation in the height of the barometer from day to day. You can construct a similar chart for yourself, from daily measurements of the height of the mercury in your siphon barometer.

Before determining this height, always move the short tube up or down until the surface of the mercury in it is level with the line drawn across the board, so that you obtain the height of the column of mercury above the surface open to the air. Find the height of your mercury column (usually referred to as the height of the barometer) and record it day by day in your note-book. You should also make a note of the state of the wind and weather each day, and so find out the weather that generally accompanies "high" and "low" barometric heights.

Readings of the Barometers during the month of.............

DAY OF MONTH.	BAROMETER READING.	REMARKS ON THE WEATHER.

The variations in the height of the barometer can be shown graphically day by day by means of squared paper (Fig. 87). The height of the barometer each day is marked by a dot, and consecutive dots are connected by a straight line. The irregular line or curve thus produced shows at a glance the variations of atmospheric pressure. Record your observations of the height of the barometer in this manner.

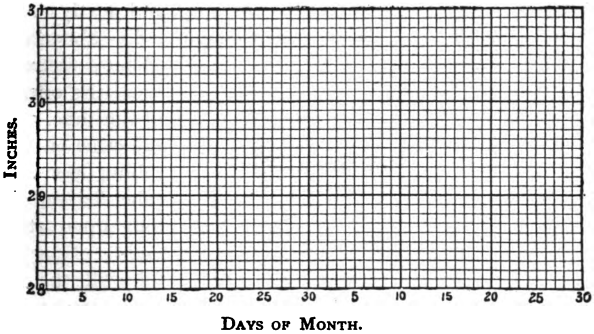

FIG. 87.—Form for recording graphically the daily readings of a barometer.

59. Variation of Barometric Height with Altitude.

Experiments have shown you that a column of mercury about 30 inches high balances a column of air extending from the surface of the mercury in the open tube to the limit of the atmosphere.

If you took your barometer up a mountain, the column of air above it would be shorter; would the pressure upon the free surface of the mercury be greater or less?

Other conditions being the same, will the mercury column of a barometer rise or fall when you ascend a mountain?

(*a*) Arrange a barometer in the lowest room available, and one in the highest, and see whether you can detect any difference in the height of the mercury columns of the two instruments at the same time day by day. As the difference will be very slight, great accuracy is required to detect it, if one room is only about fifty feet above the other.

60. Relation between Pressure and Volume of Gases—Boyle's Law.

(*a*) Select a glass tube about 20 cm. long, and neatly closed at one end (Fig. 88 *A*). Tie a piece of stout india-rubber tubing about a metre long upon the open end of the tube, and fix the other end of the tubing upon a glass tube, *B*, about 20 cm. long open at both ends. Now carefully fill the tubes with mercury until the level of the liquid is about 10 cm. from the open end. Afterwards fix the closed tube upright in a retort stand with the sealed end upwards, place your finger upon the open end of the other tube, and lower the open end so as to make the air pass into the closed tube. This apparatus will enable you to measure the expansion and compression of air.

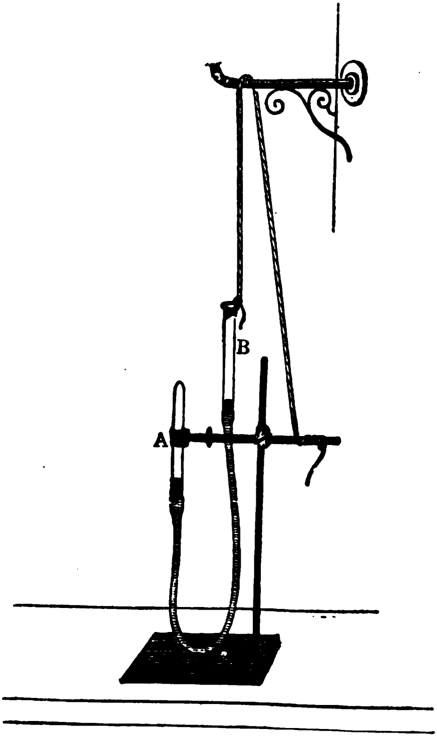

FIG. 88.—To show the relation between the pressure and volume of a gas.

Support the open tube with its open end upwards, and at such a height that the mercury stands at the same level in the closed and open tubes. The imprisoned air is then at the same pressure as the air outside.

If the closed tube is uniform in bore, and the inside of the sealed end is nearly flat, the volume of the imprisoned air is proportional to the length of tube occupied by the air, so that if the air is made to occupy one-half the original length of tube its volume is one-half the original volume. The pressure upon the enclosed air is equal to the pressure due to the column of mercury between the level of mercury in the closed tube and that in the open tube, *plus* the pressure of the atmosphere. Observe the height of the barometer, and make the difference of level of the mercury in the two tubes equal to it by lifting the open tube. Then record in your note-book as follows:

Height of the barometer, - - - - ...... cm.
Length of air column when the mercury is at the same level in both tubes, that is, when the imprisoned air is at atmospheric pressure, ... cm.
Height of barometer ...... cm. + equal height in tube, - - - - - - - ...... cm.
Length of air column under pressure of two atmospheres, - - - - - - ...... cm.

The pressure to which the enclosed air is subjected in the second case is double that of the first case; find the proportion in which the volume of air, represented by the length of air column, has been diminished.

(*b*) Lower the open tube until the air in the closed tube almost reaches the india-rubber junction. Measure the length of the air column, and the difference of level of the mercury in the two tubes. Repeat the experiment by reading the volume of air and the head of mercury at every few centimetres up to the highest point you can raise the open tube. Record your results as indicated below:

HEIGHT OF BAROMETER.	DIFFERENCE OF LEVEL OF MERCURY.	TOTAL PRESSURE ON THE AIR, P.	VOLUME OF AIR, V.	VOLUME × TOTAL PRESSURE, $P \times V$.
CM.	CM.			

Notice the numbers in the last column, representing the product of the volume of a gas and the pressure ; they should be the same for each set of observations.

A convenient and more efficient arrangement for determining the relation between the volume and pressure of a gas is represented in Fig. 89. A burette, *A*, is fixed to an upright stand by means of wire or small brass clamps. Upon the lower end a piece of thick india-rubber tubing is firmly tied. The tubing is also tied upon one end of a three-way piece, *B*. A long glass tube, *C*, connected with another end of the three-way junction is supported upon the stand side by side with the burette. To the third end of the junction a long piece of rubber tubing, *D*, is fixed, and at the top of this a large funnel or bottle, *E*, with the bottom cut off, is tied. The use of this apparatus will be seen by performing the following experiments :

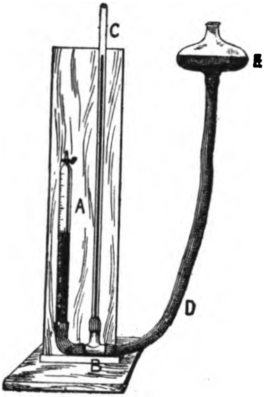

FIG. 89.—Apparatus to measure exactly the volume of air under different pressures.

(*c*) Open the stop-cock of the burette. Pour mercury into the bottle until its level is about one-third of the distance up the burette. This mercury is at the same level in the burette, long glass tube, and rubber tube, and the air in the burette is therefore at the pressure of the atmosphere. Now close the stop-cock. You have enclosed a given volume of air, which is indicated upon the burette. Pour mercury into the bottle until the bottle is nearly half full. By lifting or lowering the bottle the level of the mercury is, of course, varied. Move the bottle until the difference between the level of the mercury in the burette and in the straight glass tube is 30 inches, or whatever is the reading of the barometer at the time of the experiment.

Measure the length of the air column which is now subjected to a pressure equal to two atmospheres, and notice that it is one-half of the original volume. That is, by doubling the pressure the volume is halved.

(*d*) Perform several experiments with the Boyle's Law apparatus, beginning with the air occupying nearly the whole length of the burette, and observing in each case (*a*) the volume occupied by the air in the burette, always waiting a few minutes before taking the reading, (*b*) the difference in level between the mercury in the closed burette and the open tube.

When the air occupies less volume than it did before the stop-cock was closed it is evidently under a pressure greater than that of the atmosphere; when it occupies a greater volume it is under a pressure less than that of the atmosphere. At pressures less than that of the atmosphere, the difference between the heights of the mercury in the open and closed tubes must be *subtracted* from the barometric height and not added to it in order to obtain the value of the pressure to which the enclosed air is subjected.

Tabulate your results thus:

DIFFERENCE OF HEIGHT OF MERCURY COLUMNS.	HEIGHT OF BAROMETER.	TOTAL PRESSURE, P.	VOLUME OF AIR IN BURETTE, V.	$P \times V$.
CM.	CM.			

It will be noticed that if the total pressure (obtained by adding the height of the barometer to the difference in level of the mercury in the burette and the open tube) is multiplied by the volume of the air the result is always the same. This relation was discovered by Boyle, and is known as *Boyle's Law*. It can be expressed by saying that when the *temperature remains the same, the volume of a gas varies inversely as its pressure*. Or, what is the same thing, *the temperature remaining the same, the product of the pressure into the volume is constant.*

CHAPTER VII.

PLOTTING OF CURVES.

61. Co-ordinates.

Suppose it was necessary for some reason to exactly define the position of any letter on a page of print; the letter s, for example, in the sixth line from the bottom, on page 105 of this book. How could we do it?

One way would be to count the number of lines from the bottom line of print, and the number of letters along the line itself from the inside edge of the print. This would really amount to measuring two lengths at right angles to one another. The two lines at right angles from which our measurements are made being the bottom line of print and the inside margin of the lines.

Any two such lines at right angles, from which measurements of this kind are made, are known as *the axes*, the horizontal line being generally called the *axis of x*, and the vertical line the *axis of y*. The point where the axes intersect is called the *origin*. Distances along the axis of *x* are *abscissae* (singular, *abscissa*), distances along the axis of *y* are called *ordinates*. The abscissa and ordinate of a point are together spoken of as the *co-ordinates* of the point.

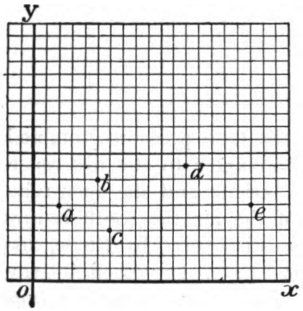

FIG. 90.—For Exercise 61 (*a*).

(*a*) Transfer the points *a*, *b*, *c*, *d*, *e* from Fig. 90 to squared paper, making the axes of

x and y coincide with two dark lines, as in Fig. 90. Using the sides of small squares as the unit of length, read off the abscissae and ordinates of each point, thus:

	Abscissa.	Ordinate.
a	2	6
b	5	8
c	6	4
d	12	9
e	17	6

(*b*) Use two of the thick lines on the squared paper as axes; take the side of a small square as the unit of length. Mark the positions of the following points.

	Abscissa.	Ordinate.		Abscissa.	Ordinate.
(1)	8	4	(5)	16	8
(2)	10	5	(6)	18	9
(3)	12	6	(7)	20	10
(4)	14	7	(8)	22	11

62. Meaning of Loci.

Notice in the last exercise that every abscissa is twice the corresponding ordinate. Observe that if you join the points (1) and (2) with a straight line, this will, if continued, pass through each of the other six points. Similarly, the line continued would pass through all points the abscissa of which was twice the corresponding ordinate. This is expressed by saying that the straight line you have drawn is the *locus* of all points the abscissae of which are twice the ordinates.

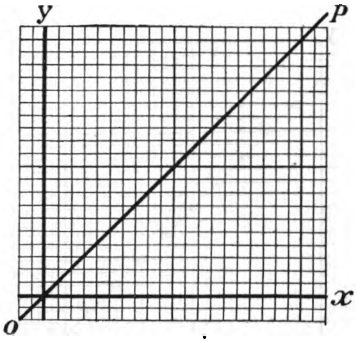

FIG. 91.—Locus of points with equal abscissae and ordinates.

(*a*) Find the locus of all points the abscissae and ordinates of which are equal. As before take two dark lines on the squared paper at right angles to one another to represent the axes of x and y. Mark in succession the points having both their abscissae and ordinates equal to 1, 2, 3, 4, 5, etc., and join these points by a line OP (Fig. 91).

(*b*) Now refer to the results obtained in Exercise 39 (*a*) where you determined the relation between the volume of water measured in cubic centimetres and its mass measured in grams. Take the numbers representing volumes as ordinates and those representing masses as abscissae. Plot these corresponding numbers on squared paper and join them as before. Notice the same sort of result is obtained as in the general case in the preceding exercise.

(*c*) Find the locus of all points the abscissae and ordinates of which, added together, always equal 12 units of length. Take a succession of values for the abscissa and calculate the corresponding value of the ordinate, thus:

	Abscissa.	Ordinate.		Abscissa.	Ordinate.
(1)	1	$12-1=11$	(6)	6	$12-6=6$
(2)	2	$12-2=10$	(7)	7	$12-7=5$
(3)	3	$12-3=9$	(8)	8	$12-8=4$
(4)	4	$12-4=8$	(9)	9	$12-9=3$
(5)	5	$12-5=7$	(10)	10	$12-10=2$

Plot out these points on squared paper as before (Fig. 92).

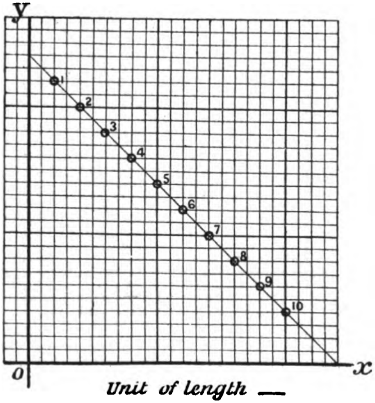

FIG. 92.—Locus determined in Exercise 60 (*c*).

(*d*) Find the locus of the points the abscissae and ordinates of which when multiplied together equal 24. Take, as before, a succession of values for the abscissae, and calculate the values of the ordinates, thus, 1-24, 2-12, 3-8, etc. Find by plotting on squared paper the locus of these points.

(*e*) Now proceed to plot the results obtained in Exercise 19 (*c*) dealing with areas measured in both square centimetres and square inches. Regard the measurement of a given area in square centimetres as an ordinate, and in square inches as an abscissa. Plot the measurements of as many areas as you have examined. Join in the points so located, and note that a straight line is obtained.

63. Symbolic Representation of a Locus.

Instead of giving different values for the abscissae of points as we have done in the previous exercises, let us use a general expression for them all, x. That is, let x stand for the distance from the axis of y measured along or parallel to the axis of x. How shall we represent each of the loci we have plotted? Let y stand for the distance of any point on the locus from the axis of x measured along or parallel to the axis of y. We proceed as follows:

To find the locus of points the abscissae of which are twice the ordinates. The abscissae are represented by x, the ordinates by y. By the terms of the problem x is always twice as great as y, or, $x=2y$, is an equation which stands for the line we plotted in Exercise 61 (*b*).

To find the locus of all points the abscissae and ordinates of which are equal. Here, as the student will at once see from Fig. 91, the equation is $x=y$.

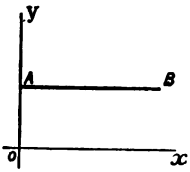

FIG. 93.

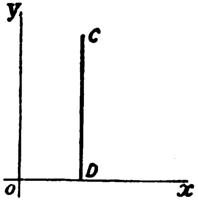

FIG. 94.

The equation of the curve obtained in Exercise 62 (*d*) is $xy=24$. Notice carefully the kind of curve which corresponds to this equation.

(*a*) Find an equation to represent the line *AB* in Fig. 93, and *CD* in Fig. 94.

64. Geometrical Representation of Varying Quantities.

We have already made use of this method of representing varying numbers in recording barometer readings. The student should refer to Fig. 87 and again examine the plan of plotting down these measurements for different days.

Construct graphic diagrams for the following cases:

(*a*) The number of 3rd class passengers by a certain popular train throughout a week:

	Passengers.		Passengers.
Monday, - -	250	Thursday, - -	220
Tuesday, - -	215	Friday, - -	185
Wednesday, -	190	Saturday, - -	235

(*b*) The amounts of the collection in pence at a church on every Sunday throughout a quarter:

	£	s.	d.		£	s.	d.
1st Sunday,	6	7	5	7th Sunday,	6	2	11
2nd "	7	10	3	8th "	5	18	1
3rd "	5	5	9	9th "	7	17	0
4th "	6	13	7	10th "	8	1	2
5th "	7	5	4	11th "	9	2	3
6th "	8	9	2	12th "	13	1	8

(*c*) The approximate population of Blackburn at the census of the following years is given in the table. As in previous exercises plot these numbers on squared paper. Join the points so obtained, and from the resulting locus read off what you would expect the population to be in 1846, 1856, 1866, 1876, and 1896.

YEAR.	POPULATION.	YEAR.	POPULATION.
1841	36,600	1871	76,300
1851	46,500	1881	104,000
1861	63,100	1891	120,100

(*d*) Repeat the preceding exercise, using the populations of the parish of St. Mary, Newington, S.E.

YEAR.	POPULATION.	YEAR.	POPULATION.
1841	54,700	1871	88,900
1851	64,800	1881	107,800
1861	82,200	1891	115,700

65. Graphic Representation of Boyle's Law.

(*a*) Plot a curve to represent your readings of volume and pressure in Exercise 60 (*d*). Let abscissae represent volume readings or those in column (*i*) of that exercise, and ordinates the pressure observations as recorded in column (*iv*).

You have a pressure reading for every volume reading or an ordinate for every abscissa. In attaching numbers to the axes of x and y on your squared paper, you must be guided by the observations you are going to plot. In this case, since no pressure reading is less than 70 cm. or 700 mm. of mercury, this will be a good value to put at the origin on the axis of y, and you can number upwards in steps of 10 cm., and so you will have 70, 80, 90, etc. marked along the axis of y. Similarly, the volume reading is never less than 8, and you may start at the origin with this figure in numbering along the axis of x.

Join in the points so located on your squared paper with a fine pencil mark. If this continuous line does not form a smooth curve, if, for instance, one point is a long way out, it will probably mean you have made a mistake in your reading of pressure or volume. If the fine pencil mark is only slightly irregular, then the irregularities show you the departure from strict accuracy of which you have been guilty in your experiments. In this case sketch in a smooth curve which shall have as many of the points on one side of it as on the other. Such a curve represents the way in which pressure and volume vary together.

Moreover, since column (v) in Exercise 60 (d) showed you volume × pressure always comes out the same product, we can say that the equation

$$p \times v = \text{some constant number}$$

represents the smooth curve we have obtained.

CHAPTER VIII.

THE LEVER, PARALLEL FORCES, AND CENTRE OF GRAVITY.

66. The Simple Lever.

(*a*) Make or obtain a lever consisting of a strip of light wood graduated in centimetres and having a thin ring screwed into one edge, above the central point, and a hook screwed into each end (Fig. 95). Hang the lever from a nail in the middle ring.

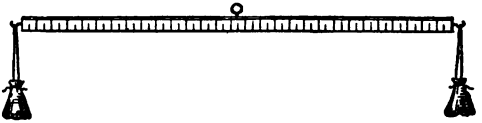

FIG. 95.—A simple lever, with linen bags suspended from the ends

If it does not exactly balance, slightly unscrew the hook at the end which rises above the horizontal, or plane a little off the end which sinks, until the lever does set itself horizontally.

(*b*) Place some pieces of lead in each of four linen bags, and adjust by means of small lead or shot until they have masses respectively of 50 grams, 100 grams, 200 grams, and 300 grams.

(*c*) Hang two pill boxes or small linen bags by their strings from the lever, one on each side of the centre, at equal distances from it. Let one of the bags contain lead or shot so that it has a mass of, say 50 grams, and place lead or shot in the other bag in order to restore equilibrium. Find, by weighing, the amount of shot required.

Repeat the experiment with the bags at a different distance.

> You will always find that equilibrium is obtained when equal masses are at equal distances from the turning point or fulcrum.

67. Principle of Moments.

(*a*) Place the 100-gram bag about 12 cm. from the fulcrum of the lever and balance it with the 50-gram bag on the other side. Record the distance from the fulcrum in each case. Repeat the experiment by balancing 50 grams against 200 grams, 100 grams against 200 grams, and 50 grams against 300 grams.

Record your observations in columns as below:

Left Side of Lever.		Load × Distance.	Right Side of Lever		Load × Distance.
Load.	Distance from Fulcrum.		Load.	Distance from Fulcrum.	

Compare the numbers in columns 3 and 6, and state in words the law or rule indicated by the results.

> The exercise shows you that there is a definite proportion between the masses on the two sides of a lever, and their distances from the fulcrum. The proportion is:
>
> Left load : Right load :: Right distance : Left distance.
>
> Or, expressed in another way, the loads are inversely proportional to their distances from the fulcrum.
>
> The turning effect of any force acting upon a lever, as each weight did in the experiment, is termed the ***moment*** of the force. The comparison of columns 3 and 6 has

shown you that the moment (load × distance) tending to turn the lever in one direction is balanced by the moment on the other side.

(*b*) Hang two of the bags on the same side of the lever at different distances and one bag on the other side. Bring into equilibrium as before. Record the positions of the bags.

Now alter the position of the single bag; bring into equilibrium and try the experiment again. Repeat several times with other bags, and compare the sum of the moments of the forces acting on one side with the moment of the force on the other side.

(*c*) Place a bag with shot or nails in it on one side of the fulcrum and an empty bag at the same distance on the other. Put weights into the latter bag until the bags are balanced. Since the arms of the lever are equal in length, the weights placed in one bag equals the mass of the shot. What is this mass?

(*d*) Hang upon the lever the bag of shot or nails used in the preceding exercise at any convenient distance from the fulcrum, and balance it with the 100-gram bag upon the other arm of the lever. From the proportion proved in Exercise 67 (*a*), you have:

Mass of shot : Known mass : : Distance of known mass from fulcrum : Distance of shot from fulcrum.

Use this proportion, or the principle of moments, to find the mass of the shot.

68. Inclined Forces acting on a Lever.

Hitherto, only forces acting at right angles to the direction of the lever have been considered. It is now necessary to discover the conditions of equilibrium when forces are inclined.

(*a*) Procure a piece of thin india-rubber cord, about a foot long. Tie loops at each end, with thread. Tie two pieces of white thread around the cord, or stick two pins through the cord, at a certain distance apart, say 10 cm. Hang a mass of 100 grams from one end of the cord and measure the distance between the pins or rings of white thread. Find also the distance between the threads when 50 grams are suspended from the cord.

Now suspend the lever from the central hook. Hang 100 grams from one arm, and put a loop of the india-rubber over the hook at the end of the other arm.

Keep the lever horizontal by holding it or fixing it, while you stretch the india-rubber. When the distance between the white threads is the same as when 100 grams were suspended from the cord, fasten the india-rubber to the table with a drawing pin, as in Fig. 96. Move the mass on the other arm of the lever until the lever remains horizontal, and while the white threads are at the 100-gram distance apart.

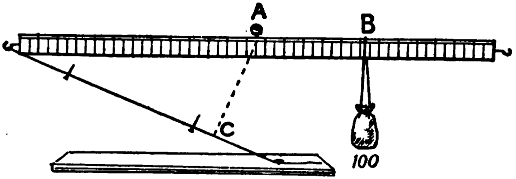

Fig. 96.—Experiment to illustrate the principle of moments when one of the forces acting on a lever is not perpendicular to the axis.

You now have the weight of the 100-gram bag acting at right angles to the direction of the lever on one side of the fulcrum, and a force equal to the weight of 100 grams acting inclined to the lever on the other side. Measure the perpendicular distance *AC* from the fulcrum to the india-rubber cord; it will be found to be the same as the distance *AB*.

(*b*) Repeat the experiment with 50 grams on one side of the fulcrum, and the cord stretched to the 50-gram length.

> It is thus seen that when a force acts upon a lever at an inclination the moment is equal to the force multiplied by the perpendicular distance from the fulcrum.

69. The Principle of Work applied to Levers.

(*a*) Hang the lever upon a nail on a wall, and when it is horizontal draw a line on the wall along the upper edge. Hang a 100-gram bag from the hook at one end of the lever, and a 300-gram bag at one-third the distance from the fulcrum to the end on the other side. The two bags will counterpoise one another. Now turn the lever so that the end of the short arm moves through 4 or 5 cm. from *B* to *b* (you can measure this distance by means of compasses or calipers). Also measure the distance *Cc* through which the end of the long arm moves. Compare the small with the larger value in each measurement, thus:

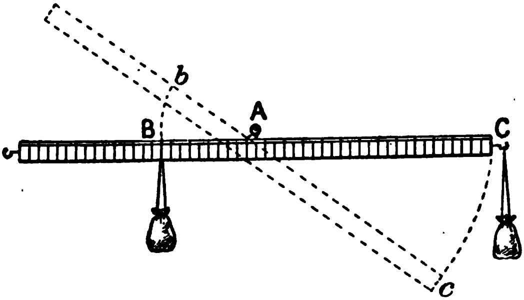

FIG. 97.—Experiment to illustrate the principle of work.

$\frac{\text{Length of Long Arm}}{\text{Length of Short Arm}}$	$\frac{\text{Length of Long Arc, } Cc}{\text{Length of Short Arc, } Bb}$.	$\frac{\text{Large Mass}}{\text{Small Mass}}$.

It will be found that the values obtained in each of these columns are practically the same, thus proving that when a force or effort exerted upon one arm of a lever moves a load or overcomes resistance at the other end, the distances through which the effort is exerted bears the same proportion to the distance through which the resistance is overcome that the resistance itself bears to the effort. In other words, what is gained in force is lost in distance, so that in each case the product of force and distance is the same whether the lengths of the lever arms or the arcs through which the ends of the lever moves are taken as the distances.

70. Steelyard and Bent-Lever Balance.

(*a*) Suspend the lever from a point a few centimetres from one end. Wrap lead around the short end until the lever sets horizontally. Suspend a bag with a 100-gram weight in it from the loaded end, and balance it with a 10-gram weight placed in a bag, at the other side of the fulcrum.

Mark upon the unmarked side of the lever the position where the 10-gram weight has to be placed in order to counterpoise the 100-gram bag.

Mark also the places where the 10-gram weight has to be when 50-gram and 20-gram are substituted for the 100-gram bag. Having found the positions in which 10 grams will balance 20, 50, and 100 grams, you will be able to estimate roughly the positions for every 10 grams up to 100.

Now put less than 100 grams of shot in a bag, and hang the bag from the loaded end of the lever.

Balance the shot with the 10-gram weight in a bag on the other side, and, by noting the position in which the bag has to be placed, estimate the mass of the shot.

Repeat the experiment with different quantities of shot in the bag on the loaded arm, and test one or two of your results by weighing the shot in your balance.

The method you employ in these experiments, when the object being weighed is always placed at a fixed distance from the fulcrum, and the weight used is that of a certain mass placed at different distances, is the same as that utilised in the *Steelyard* (Fig. 98).

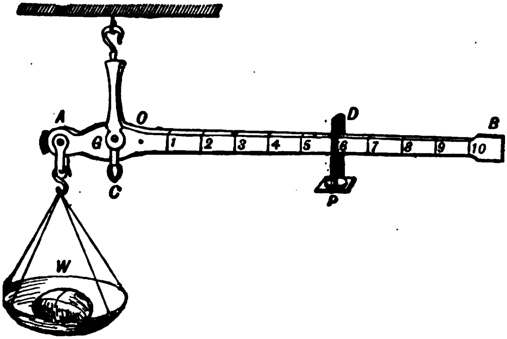

FIG. 98.—A steelyard.

(*b*) Unscrew the ring from which the lever has hitherto been suspended, and support the loaded lever in a loop of stiff wire instead. Suspend 100 grams in a bag from the unloaded end, and move the loop of wire until the lever balances in it. Mark the position of the wire loop or fulcrum.

Repeat the experiment, using 50, 20, and 10 grams in the bag instead of 100 grams. Mark the position of the fulcrum in each case.

Notice that the marks are at different distances from each other. Now hang a bag containing shot or nails from the un-

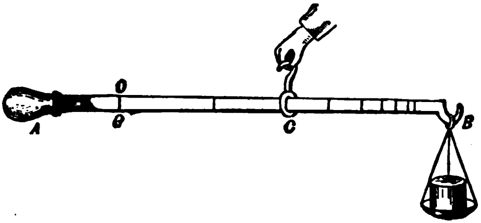

FIG. 99.—A Danish steelyard.

loaded end, and estimate its mass by noting the position of the fulcrum when it is counterpoised.

You will thus see that, by properly graduating a lever with a loaded end, it is possible to determine the mass of an object by placing it at the opposite end, and noting the position of the fulcrum when the lever is horizontal. This is the principle used in the *Danish Steelyard* (Fig. 99).

(*c*) Observe the construction of the bent-lever balance such as is used for weighing letters (Fig. 100). Adjust the balance until the index points to zero when there is nothing in the pan. Test the accuracy of the divisions on the divided arc, by placing different weights in the pan and noting the values indicated upon the graduated arc.

This balance is an interesting application of the principle of the lever. The masses on the two sides of the fulcrum are the constant mass *B* and the unknown mass on *A*. The direction of action of the force due to *B* is vertically downward, but the distance of this force from the fulcrum differs according to the position of *B*, being least when *B* is at its lowest point and greatest when *B* is at right angles to the support. The force due to the mass in the scale pan *A* also acts at varying distances from the fulcrum, and is at its greatest distance when the crank carrying the rod to which *A* is fixed is at right angles to the vertical upright.

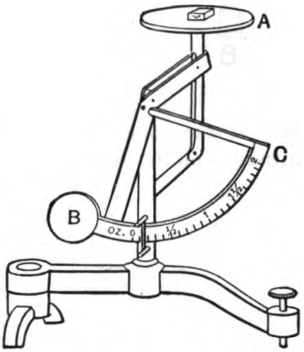

FIG. 100.—Bent-lever letter balance.

71. Resultant of Parallel Forces.

(*a*) Suspend the light lath previously used for a lever, from a spring balance by the ring above its centre. Notice the reading of the balance. Hang two *equal masses* in bags from the ends of the lath, and again observe the reading of the spring balance.

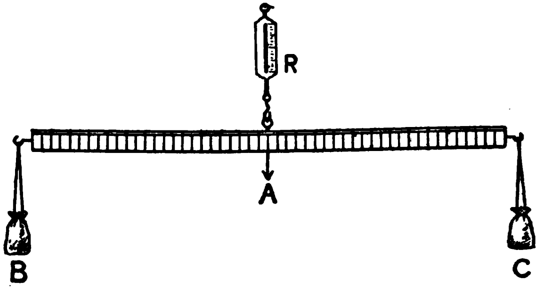

Fig. 101.—Parallel forces in equilibrium.

Repeat the experiment with *unequal masses* placed in bags arranged on the lath so as to counterpoise one another.

Mass of Lever, A.	Mass, B.	Mass, C.	Total, $A+B+C$.	Reading of Spring Balance, R.

By comparing columns 4 and 5 it will be seen that the three downward forces A, B, and C, acting upon the lath, are kept in equilibrium by one upward force D.

(*b*) Hang the two masses B and C in one bag from the middle of the lath suspended from the spring balance. Notice that the reading of the balance is the same as when the two masses are hung from the ends of the lath.

When a single force can be substituted in this way for two or more separate forces without affecting the equilibrium of the body acted upon, it is said to be a ***resultant***, and the separate forces are termed ***components***.

72. Conditions of Equilibrium of Parallel Forces.

To further study accurately the conditions of equilibrium of parallel forces, a method is required which does away with the necessity of considering the mass of the lever. This is obtained as follows:

(*a*) Suspend the lath by a string which passes over a pulley and has attached to the other end a load equal to the mass of the lath (Fig. 102). The rod can then move as if it had no mass.

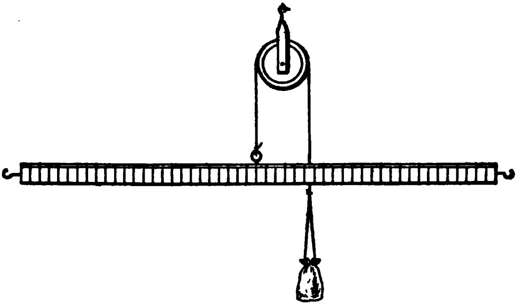

FIG. 102.—Lever counterpoised for experiments with parallel forces.

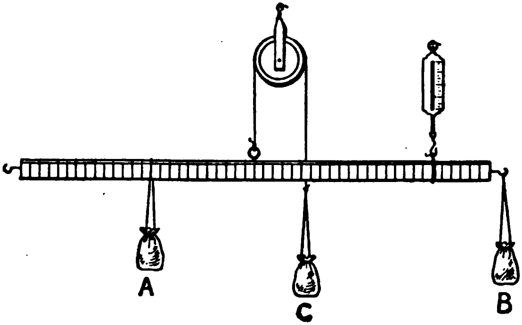

FIG. 103.—Parallel forces in equilibrium.

Upward forces can be applied to it by spring balances, or masses attached to strings passing over pulleys and downward forces by hanging masses from it.

(*b*) Attach a spring balance to any convenient point on the lath by passing the hook of the balance through a hole in the lever or by means of thread. Suspend masses *A* and *B* from the lath so that they counterpoise one another. Notice reading of spring balance and masses used.

Add an additional mass to that used to counterpoise the lath, and also add masses to the lath to restore equilibrium.

MASS, *A*.	MASS, *B*.	TOTAL DOWNWARD FORCES, WEIGHT OF $A+B$.	WEIGHT OF ADDITIONAL MASS, *C* = UPWARD FORCE.	DOWNWARD FORCES − UPWARD FORCE.	READING OF SPRING BALANCE, RESULTANT.

A comparison of columns 5 and 6 will show you that the pull on the balance is equal to the difference of the forces acting downwards and upwards respectively. The reading of the balance shows, in fact, the value of the resultant of all the forces acting upon the lath.

(*c*) Remove the masses, except the one counterpoising the lath. Select a mass the weight of which is equal to the previous resultant reading of the spring balance. Hang it from the lath, and move it about until the reading of the spring balance is the same as before.

You can thus prove that the direction of the resultant force passes through the fulcrum formed by the supporting hook of the spring balance.

73. Equilibrium of Three Parallel Forces.

(*a*) Using the same arrangement as before, suspend masses from the lath and move the spring balance along the lath until they counterpoise one another. Repeat the experiment with different loads, and observe in each case (*a*) the reading of the spring balance, (*b*) the distance of each mass from the point at which the spring balance is attached to the rod, (*c*) the total distance between the masses. These observations can be used to prove several important points in connection with parallel forces in equilibrium (Fig. 104). To begin with, the numbers should be arranged thus:

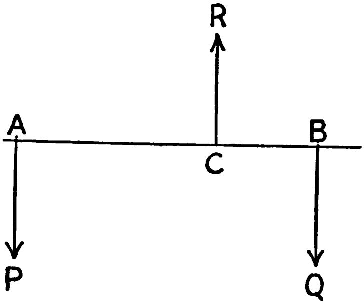

FIG. 104.—Representation of parallel forces in equilibrium.

MASS, *P*.	MASS, *Q*.	DISTANCE, *AC*.	DISTANCE, *BC*.	$P \times AC$.	$Q \times BC$.

A comparison of columns 5 and 6 will show that one mass multiplied by its distance from the point of action of the resultant is equal to the product of the other mass by the other's distance from the resultant. Or, as has already been proved with the simple lever:

$$\left.\begin{array}{c}\text{Sum of moments on one}\\ \text{side of fulcrum}\end{array}\right\} = \left\{\begin{array}{c}\text{Sum of moments on other}\\ \text{side of fulcrum.}\end{array}\right.$$

(*b*) Three parallel forces were used in the preceding experiment, and three distances were measured. They are:

Force *R*, Distances *AB*
„ *P*, „ *AC*
„ *Q*, „ *BC*

Select each force in turn and divide it by the distances between the two other forces, thus :

$\frac{\text{Force } R}{\text{Distance } AB}$.	$\frac{\text{Force } P}{\text{Distance } BC}$.	$\frac{\text{Force } Q}{\text{Distance } AC}$.

From the results you see that when three parallel forces are in equilibrium each force is proportional to the distance between the two other forces.

(*c*) Calculate the moments of R and Q of Fig. 104 about the point A ; they are $R \times AC$ and $Q \times AB$. Also calculate those of P and R about B and of P and Q about C; what do the results show?

Select any point in AB, say X. Calculate the moments $P \times AX$, $R \times CX$, and $Q \times BX$. Give a negative sign to those which tend to turn the bar in the direction in which the hands of a clock move, and a positive to those with opposite effect, and then find the actual algebraic *sum* of the moments about the point X.

74. Determinations of Centre of Gravity.

Uniform Rod.

(*a*) Obtain a uniform rod of wood—an uncut lead pencil will do—and find at what point a loop of thread tied upon it must be placed in order that the pencil will balance.

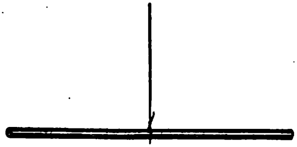

FIG. 105.—Pencil supported by a thread in the same vertical line as the centre of gravity.

Circular Disc.

(*b*) Procure a circular disc cut out of stout cardboard and having several holes bored near the circumference. Make a plumb-line with a thread having a piece of lead tied to one end

To the other end tie a bent pin or a hook of thin wire. Hang the disc from the hook, as shown in Fig. 106, and let the plumb-line hang in front of it. (The disc should swing easily upon the hook.) Mark the direction of the plumb-line upon the cardboard, which may be done by chalking the thread, fixing it at two points with the finger and thumb and drawing it away from the disc and releasing it like the string of a bow. Repeat the experiment with the cardboard suspended from another hole. Notice the point (known as the *centre of gravity*) where the two lines intersect. Make a pin-hole at this point and pass a thread knotted at one end through it. Hold up the cardboard by the thread and see whether it sets horizontally.

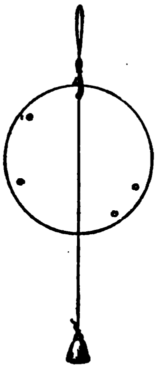

FIG. 106.—Vertical disc hung from a hook, for the determination of centre of gravity.

Parallelogram.

(*c*) Repeat the preceding experiment with a cardboard parallelogram instead of a disc, in this case suspending it from three different points. After you have drawn the three directions of the plumb-line, turn over the cardboard and draw the two diagonals upon the opposite side. Make a pin-hole where the diagonals intersect. Is the point where the diagonals intersect the same as that found practically?

(*d*) Take a cardboard parallelogram of which you have not found the centre of gravity. Mark where the plumb-line cuts the lowest edge and join this point to the point of suspension. Repeat with the parallelogram suspended from another point. Each line passes through the centre of gravity. Mark the point where they act, and try by balancing whether this point is actually the centre of gravity.

Triangle.

(*e*) Repeat the preceding experiment, with a triangle cut out of cardboard. After finding the centre of gravity by means of the plumb-line, prick a pin-hole through the cardboard, then turn over the triangle and draw a line from each angle through the pin-hole to the opposite edge. Now, take each edge in turn as the base of the triangle, determine (1) the lengths of the two parts into which each base is divided by the lines drawn, (2) the proportion which the distance of the hole from each base bears to the length of the line from that base to the opposite angle.

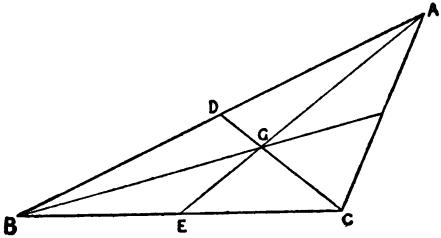

Fig. 107.—Centre of gravity of a triangle.

If the centre of gravity has been accurately found, each base will be divided into equal parts, and the distance of the hole from each base will be one-third the distance from the base to the opposite angle (Fig. 107).

Quadrilateral.

(*f*) Cut a four-sided figure out of cardboard, and draw a line connecting two of the opposite angles. Find by bisecting this line and taking $\frac{1}{3}$ the distance from the middle to the opposite angle, the centre of gravity of each of the triangles into which the figure is divided. Connect the two points found. Then draw the other diagonal of the quadrilateral, repeat the measures, and connect the centres of gravity as before. Test whether the point where this short line cuts the other is the centre of gravity of the whole figure.

Irregular Figure.

(*g*) Select a piece of cardboard of any shape, and find its centre of gravity. Test, as before, whether the cardboard will set horizontally when suspended from the point found.

Open Framework.

(*h*) Procure a skeleton cube or tetrahedron, and suspend it as in the preceding experiments. Mark the verticals through the point of suspension by light wires attached by wax, and thus find the position of the centre of gravity.

(*i*) Find the centre of gravity of an open wicker-work basket, such as a waste-paper basket. To do this, suspend the basket, and hang a plumb-line from the point of suspension. Tie a piece of thread across the basket in the direction of the plumb-line; then suspend the basket from another point, and notice where the plumb-line crosses the thread. The point of intersection is the centre of gravity; it need not be actually a point on the framework itself.

75. Centre of Gravity in Relation to Equilibrium.

(*a*) Hang one of the cardboard figures used in the preceding experiments loosely from a pin stuck into a wall or upright board. Notice the direction of the line connecting the centre of gravity with the point of support, when the cardboard comes to rest. Try the experiment with the pin at different distances from the centre of gravity. Record your observations, and state in your own words what conditions with reference to centre of gravity and point of support must be fulfilled in order that an object shall be in equilibrium.

(*b*) Place one of the pieces of cardboard, previously used, near the edge of a square-edged table, and push it gently until it would just topple over if permitted. When this condition is obtained, mark upon the under side of the card where the edge of the table comes. Repeat the experiment with the cardboard in a different position, and again draw a line on the under side. Test whether the point of intersection of these lines marks the centre of gravity.

(*c*) Procure an oblong block of wood of about the same size as this book, and about three inches thick. Draw the two diagonals on one of its faces; the centre of gravity of the block will be inside the block at a point below the intersection of the diagonals. Press a pin partly into the block at the point where the diagonals intersect, and suspend from it a small plumb-line (Fig. 108). Now place the block upon a board, and notice the direction of the plumb-line. Tilt the board until the block topples over. (To prevent the block from sliding, it should rest against a couple of small nails fixed in the board.) When the block topples over, the plumb-line and the centre of gravity, which it indicates, will be seen to fall beyond the base.

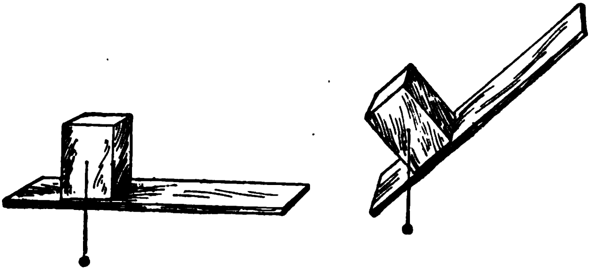

FIG. 108.—Relation between centre of gravity and the base of an object.

76. Simple Pulley, Fixed.

A simple pulley can be constructed as follows: Obtain a grooved wheel, such as is fixed to one end of a blind-roller. Fill up the centre with a plug of wood or cork, and at the centre of the plug bore a hole and fit a short piece of glass tubing firmly into it. Pass a piece of thick brass wire through the tube,. and then bend it as in Fig. 109. Another plan is to pass a piece of a knitting needle through the tube and bend the ends of the brass wire into loops upon which the needle rests. But aluminium pulleys, which can now be obtained at a moderate cost, are best for the following exercises.

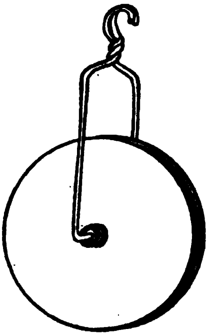

FIG. 109.—A simple pulley.

(*a*) Hang a pulley from a stand or nail, and over the wheel or sheaf pass a fine flexible cord having a loop tied at each end. Connect a light tray or bag with each end of the cord. Put a weight in one of the trays and then gradually place bits of lead or small nails in the other until the first tray moves (Fig. 110). When this happens, take each of the trays and weigh them with their contents. Repeat the experiment with different weights and record your results in parallel columns.

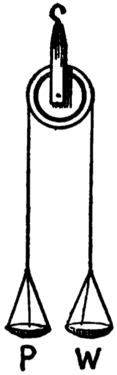

FIG. 110.—Use of a single fixed pulley.

The two trays with their contents will be found to weigh very nearly the same, but not exactly, for in order to move the tray *W*, the mass *P* must be a little greater than that of *W*, as the friction of the pulley upon its axle has to be overcome.

The only advantage of using a fixed pulley, as in the preceding experiment is, that a mass can be moved in one direction by a force exerted in another. When, however, a pulley, instead of being fixed in position, can move up or down, it is possible to lift a mass by a smaller effort than would otherwise be necessary.

77. Tension in String Supporting Movable Pulley.

(*a*) Place a mass in one of the trays previously used, and suspend the tray and a pulley together from a spring balance. Record the reading of the balance. Now arrange the pulley and balance as shown in Fig. 111, and again record the reading.

MASS W.	READING OF BALANCE, P.	$\frac{P}{W}$

It will be noticed that P is only $\frac{1}{2}W$ (roughly) in every case; in other words, the tension in a cord supporting a movable pulley having a mass hung from it is equal to one half the weight of the total mass supported. This result might have been predicted from what you have learned of parallel forces.

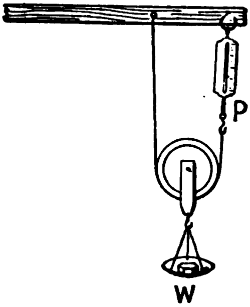

FIG. 111.—Experiment to show that the tension in a string supporting a mass from a single movable pulley is equal to half the weight of the mass supported.

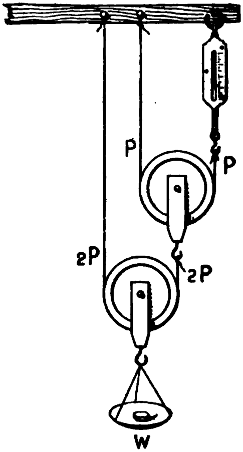

FIG. 112.—Experiment with two movable pulleys.

(*b*) Hang the tray with weights in it, together with two pulleys, from a spring balance as before, and observe the reading. Then arrange the pulleys in connection with the spring balance as shown in Fig. 112, and again observe the reading. Tabulate your observations.

MASS W.	READING OF BALANCE, P.	$\frac{P}{W}$

In this case, that is, with two movable pulleys, it will be noticed that the force required to support the pulleys and included mass is less than before. Consider this experiment from a theoretical point of view. The upper pulley is supported by two cords, in each of which the tension is P. If the pulley had no mass the single force required to counteract this tension would be equal to $2P$, but taking the pulley into account, the force is $2P - W_1$, where W_1 represents the weight of the pulley. The tension in the string around the lower pulley is thus equal to $2P - W_1$, and if the pulley had no mass the load W would be equal to $2(2P - W_1)$. But taking the mass (W_2) of the second pulley into account it is found that W is equal to $2(2P - W_1) - W_2$. In the same way the relation between the load and sustaining force for any number can be determined. To test how far this statement is correct, perform the following experiment:

(*c*) Suspend an unknown load from a pulley, the mass (W_1) of which is known. Observe the force $2P$ required to support the load. Then calculate the value of $2P - W_1$ and compare it with the load, found by weighing, which was supported. Repeat the exercise with two pulleys, in this case comparing the load with the value of $2(2P - W_1) - W_2$, where W_2 represents the mass of the second pulley.

78. Mechanical Advantage.

Another way to determine the advantage of using movable pulleys is to pass the free end of the cord over a fixed pulley, as in the following experiments, instead of connecting it with a spring balance.

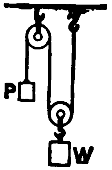

FIG. 113.—A fixed and a movable pulley.

(*a*) Arrange one fixed and one movable pulley as in Fig. 113. Place various masses in succession in the tray W, suspended from the movable pulley, and add lead or shot to the tray P until W begins to move. Then disconnect the pulleys and find the mass of P and its contents, and the total mass of W and the movable pulley. Tabulate your observations thus :

TOTAL MASS LIFTED (W+PULLEY).	MOVING FORCE, P.	$\frac{\text{LOAD}}{\text{MOVING FORCE}}$

In mechanics the number of times the load moved or resistance exceeds the force used or effort exerted is known as the *mechanical advantage*. The mechanical advantage obtained by using one movable pulley is therefore shown in the third column.

79. Principle of Work applied to Pulleys.

(*a*) Arrange a fixed and a movable pulley as in the preceding exercise. Measure the distance of the bottom of P from the table, and also the distance of the bottom of W from the table. Then move P through a certain distance and measure again the distances of P and W from the table. It will be found that P has moved through about twice the distance by which W moves. Compare the products, P × distance moved and W × distance moved.

CHAPTER IX.

MEASUREMENT OF TIME AND ANGLE.

80. The Simple Pendulum.

(*a*) Tie a small leaden ball, with hook attached, to a piece of thread, and fix the free end of the thread to a suitable support. Set the ball oscillating, but do not let it swing very widely. Ascertain the time taken for the ball to make twenty oscillations. You can easily manage this if you have a stop watch, and you allow the ball to swing in front of a piece of white paper on which a distinct line is drawn where the ball hangs when at rest. In counting the oscillations, notice when the ball swings past the mark going in the same direction, that is, count an oscillation as a complete movement to and fro of the pendulum.

Repeat the experiment several times, recording your results thus:

Time taken to complete 20 oscillations—
1st experiment = ... seconds.
2nd ,, = ... ,,
3rd ,, = ... ,,
4th ,, = ... ,,
Mean = ... ,,

Time t to complete one vibration = ... seconds.

(*b*) Measure the length from the point of suspension of the thread to the centre of the ball. Do this in two steps. First measure the length of the cord from the point of suspension down to the ball by placing a scale by the side when the ball hangs freely. Secondly, measure the diameter of the ball by means of the callipers or screw-gauge, and divide the result by 2 to obtain the radius of the ball. Add these two results together and so obtain the length l of the pendulum.

Alter the length by shortening the thread, again measure the time of swing and the length. Do this three or four times.

Arrange the results in this and the last experiment as below:

TIME $=t$ SECONDS.	$t\times t=t^2$.	LENGTH l CENTIMETRES.	$\frac{l}{t^2}$.

The quotient obtained in the last column is a constant number, and shows us that the square of the time of oscillation of a simple pendulum is proportional to its length.

(*c*) To further test this statement compare your results thus :

NUMBER OF SWINGS (n) IN 10 SECS.	n^2.	LENGTH OF PENDULUM (l).	n^2l.

Observe that the product of the square of the number of vibrations in a given time by the length is constant; or what is the same thing, the square of the number of oscillations is inversely proportional to the length.

(*d*) Draw the pendulum ball aside, say ten inches from the vertical, and find the times required to complete twenty swings. Then draw it only three inches aside and again find the time of twenty vibrations. Hence determine whether the time is independent of the amplitude.

(*e*) Determine the time required for a certain number of swings when balls of the same size and shape, but different masses, are suspended from the same length of thread. It will be found that within certain limits the mass of the body does not affect the rate of swing.

(*f*) Calculate according to the rule as to length of pendulum found in Exercise 80 (*b*), the length of a pendulum which will perform one complete vibration in a second. Make a pendulum of the calculated length and test whether it actually beats seconds. If not, adjust it until the time of one oscillation is a second, and then measure the length of the pendulum.

81. Vibrations of a Thin Flexible Lath.

(*a*) Fix one end of a lath about 4 or 5 feet long and $\frac{1}{4}$ inch thick in a vice. Pluck the other end, and observe that it swings backwards and forwards. Find how long in seconds it takes to make 20 swings, and calculate the rate of swinging. Test if within certain limits the rate of swinging is the same whether the swings be great or small.

(*b*) Fix the rod as before and count the number of swings in 20 secs., and measure the length of the vibrating part. Alter the length so as to make the rod shorter; again count the number of swings and measure the length. Do this three or four times until the vibrations become too rapid to measure. Record thus:

LENGTH (l)	NUMBER (n) OF SWINGS.	l^2.	$n \times l^2$.

Observe that the product nl^2 is approximately constant, that is, the number of vibrations is *inversely* proportional to the square of the length, as was found to be the case with a pendulum.

(*c*) Obtain a rod of rectangular section—a convenient size is 6 feet × $\frac{1}{2}$ inch × $\frac{1}{4}$ inch. Clamp the rod in the vice, so that a pair of opposite faces are horizontal. Observe the number of swings in about twenty seconds. Now turn the rod round in the vice, so that the other pair of faces is horizontal, and again observe the number of swings in the same time. In each case measure the distance between the horizontal faces, which is the thickness of the rod. Record your results:

THICKNESS OF ROD.	NUMBER OF SWINGS IN 20 SECONDS.	RATIO OF SWINGS.	RATIO OF THICKNESS.

Columns (3) and (4) will be found to be the same, thus demonstrating that the number of vibrations varies directly with the thickness of the rod.

82. Measurement of Angles.

(*a*) Draw two lines at right angles to one another. Push a pin firmly into the point *O*, where the lines cross or intersect. Tie a loop at each end of a short piece of strong thread. Put one loop over the pin and a pencil in the other, and, keeping the thread tight, draw a circle with the pencil in the loop (Fig. 114).

How many right angles are there facing the circumference of the circle?

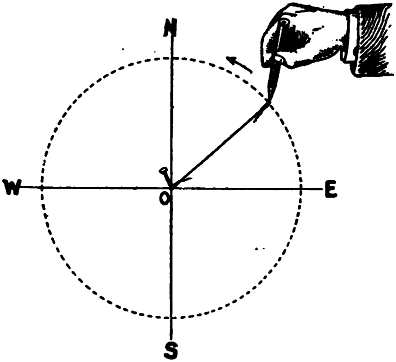

FIG. 114.—Angle produced by the rotation of a line.

(*b*) Draw a smaller circle with the same point *O* as centre. Does this circle contain the same number of right angles?

Does the size of an angle depend at all upon the lengths of the lines containing the angle?

(*c*) Place the thread in the direction *OE*, then move it to the direction *ON*. Through what angle has the thread been turned? Continue moving the thread until it lies in the direction *OW*. Through how many right angles has it now been rotated? Take the thread to the position *OS*; it has now been turned through three right angles; and if it is brought again to the direction *OE*, it will have been rotated through four right angles.

The thread in the preceding exercise was taken through a complete rotation, and brought back to its original direction in four steps, the angle between two successive directions being a right angle. If 90 equal steps had been taken in passing from the direction *OE* to *ON*, and a line had been drawn to show the direction of each position of the thread, the right angle *EON* would be divided into 90 small angles.

(*d*) When a thread is taken completely round the circle in steps of this kind, how many steps are required to bring it back to its original position?

If a right angle is divided into 90 equal parts, how many of those small angles would make up four right angles?

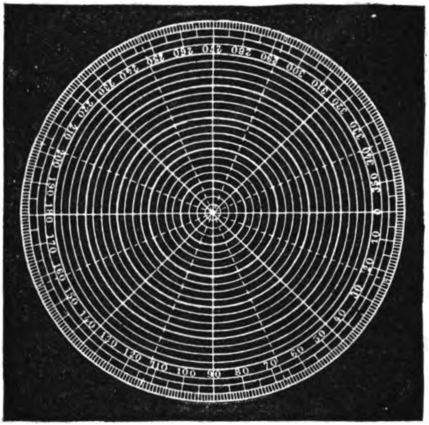

FIG. 115.—A circular protractor, to illustrate the measurement of angles.

These small angles are known as *degrees*, and are denoted thus °. One sixtieth ($\frac{1}{60}$) of a degree is one *minute of arc*, and written thus 1′; and one sixtieth of this is one *second of arc*, written thus 1″. (It is unfortunate that the signs ′ and ″ are also used to denote feet and inches.)

(*e*) If lines are drawn one degree (1°) apart, from the centre of a circle all around the circumference, into how many parts is the circumference divided?

(*f*) By means of Fig. 115, or a protractor, draw lines inclined 30°, 45°, 60°, and 120° to one another.

> To do this by means of Fig. 115, the paper upon which the lines are to be drawn should be placed under this page. A small prick with a pin should then be made at the centre of the circle, one at the point 0 and a third at the point where the number of degrees required is marked upon the circumference of the circle. The drawing paper is then taken out, and lines are drawn to connect the three pin pricks.

(*g*) Add together, by construction, angles of 15°, 25°, and 50°. Measure the total angle thus constructed.

(*h*) Add together, by construction, angles of 30°, 15°, 60° 45°, 20°, and 10°, and measure the total angle.

(*i*) Construct an angle of 100°; subtract from it, by construction, an angle of 55°.

(*j*) Subtract, in succession, from an angle of 150°, angles of 60°, 15°, and 30°.

CHAPTER X.

PARALLELOGRAM AND TRIANGLE OF FORCES, FRICTION, AND INCLINED PLANE.

83. Relation between Tension and Extension.

(*a*) Clamp one end of a piece of thin india-rubber cord, about 20 cm. long, in a retort stand or suspend it in any other convenient way. Clamp a wire or a strip of wood about the same length with it, so that the lower end is a little above the lower end of the cord. Tie a loop of strong thread to the bottom of the cord, and hang from it a light tray (Fig. 116).

Push a pin through the cord so that it is level with the bottom of the wire or wood.

Place 10 grams in the tray and measure the elongation produced by it, that is, measure the distance between the pin and the bottom of the wire or wood. Place another 10 grams in the tray, and again notice the extension produced. By subtracting this reading from the former one, you can find the extra extension produced by the additional 10 grams. In the same way find the extension produced by 30 grams, 40 grams, and so on up to 100 grams, also determining in each case the extension for the additional 10 grams. Record thus:

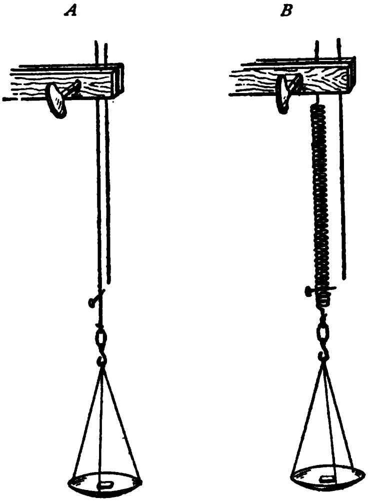

FIG. 116.—Experiment to determine relation between tension and extension: *A*, india-rubber; *B*, spiral spring.

LOAD.	TOTAL EXTENSION.	EXTENSION FOR 10 GRAMS.

Your observations will have shown you that the extension of the india-rubber per 10 grams is not quite constant. More exact results can be obtained by using a spiral spring, which can be made by tightly and closely winding a thin brass or steel wire round a thin rod until a coil a few centimetres long is obtained. The ends of the wire are twisted into loops for the purpose of suspension. A simple spring balance, consisting of a coil of this kind mounted upon a graduated strip of wood, can be obtained from many scientific instrument makers. An instrument of this kind, used for the measurement of *forces*, is called a *dynamometer*.

(*b*) Using a spiral spring, measure, as you have already done with the india-rubber, the extension produced by different loads. After every observation, take out the load from the tray, and let the spring go back to its original length. Record the observations as before.

84. Graphic Representation of Forces.

(*See also pp.* 104-110.)

(*a*) Draw vertical lines to represent the extensions produced

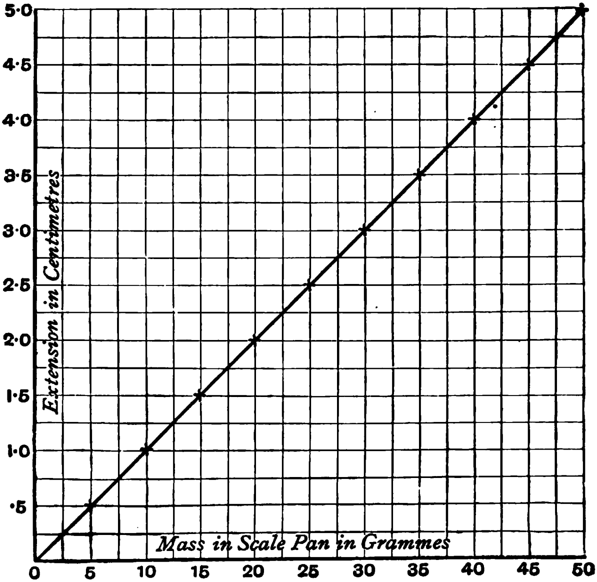

FIG. 117.—Extensions produced in a spiral spring by different loads. From Earl's *Physical Measurements* (Macmillan).

by the various loads used in the preceding experiment, and also lines to represent the extension for every additional 10 grams.

> As the weights of the loads represent forces, the lines will represent the extension produced by various forces. You will find that the extensions are proportional to the forces, that is, double the force double the extension, and so on. *The strength or magnitude of a force can thus be shown by the length of a line.*

(*c*) To show the relation between extension and the force used, plot your results as shown in Fig. 117, which represents the extension of a spiral spring a centimetre in diameter by loads up to 50 grams. Notice that the line connecting the points of observation is a straight one, thus showing that the extension of the coil is exactly proportional to the load, that is, to the force with which the spring is pulled.

85. The Principle of the Parallelogram of Forces.

> The following experiments illustrate how the *direction* as well as the *magnitude* of forces can be represented by lines.

(*a*) To the middle of a piece of elastic cord about six inches long fasten a piece of the same cord about three inches in length. From the point where the three cords meet measure off, along each cord, an equal distance, say two inches, and put a pin through each cord at this distance.

Fasten the pin at each end of the six inch piece of cord upon a sheet of paper lying flat upon a drawing board or table. The cord should lie in a straight line between the pins, but it must not be stretched (Fig. 118 i.). Now pull the third cord in any direction so as to stretch the others as well as itself, and fasten its pin in the board while the cords are thus stretched (Fig. 118 ii.). Make a pin mark upon the paper at the point where the cords meet, and then pull the pins out of the paper and put them and the cords aside. Pin-holes will be marked at *ABCD* in

Fig. 118 iii. Draw the lines *AC*, *BC*, *CD*, and cut off *AE*, *BF*, and *DG* equal to 2 inches. *CE*, *CF*, and *CG* will then represent forces. Construct the parallelogram *ECFH*. Draw the diagonal *CH*. If you have done the experiment with care, the diagonal *CH* will be found to have the same length and lie in the same line as *CG*.

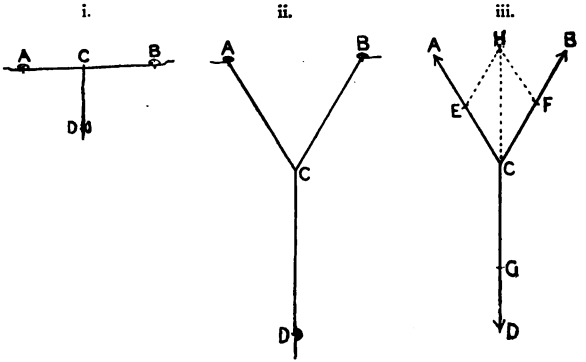

FIG. 118.—(i.) Three-way india-rubber cord; (ii.) stretched cords; (iii.) parallelogram showing forces in action.

(*b*) Now hang the cord from the place where the three pieces meet. Suspend a tray from each, one after the other, and place masses in it until the length of the part of the cord under observation is the same as the corresponding length when stretched upon the board. Weigh the tray and contents for each case. You thus find the force (which is equal to the weight of each of the masses placed in the trays) required to stretch the cord to the corresponding length of the side or diagonal of the parallelogram. Write your results against the sides or diagonal of the parallelogram just constructed.

> If you are sure that the extension of the cord you use is exactly proportional to the force, it is not necessary to repeat these weighings in other cases.

(*c*) Repeat the experiment with the cords stretched by different amounts and in different directions. For instance, arrange the cords so that the angle *ACB* is a right angle, and also make the construction when the angle *ACB* is 60° and 120°. Keep each of the parallelograms constructed, and mark the lengths of *CE*, *CF*, *CD*, and *CH* in every case.

When the cords are arranged in any position they represent three forces acting upon the particle *C*. Since *C* is at rest, the force represented by *CG* produces the same effect as the two forces *CE*, *CF*. If, therefore, the forces *CE*, *CF* were removed, and a force equal to *CG* acting in the opposite direction were put in their place, the particle *C* would still remain at rest. The diagonal *CH* represents such a force. When one force can be substituted for two or more in this way, it is called the *resultant* of the forces it replaces. Your experiments have shown you that the resultant of two forces acting in directions represented by two adjacent sides of a parallelogram is equal, both in magnitude and direction, to the diagonal of the parallelogram included between these sides.

Instead of showing that a force can be represented by parallelograms, you can perform the converse experiment and prove that any parallelogram can be regarded as representing certain forces.

(*d*) Stick the pins at the ends of the six-inch cord into a sheet of paper on a flat surface, taking care not to stretch the cord. Remove the pins, and draw any parallelogram having the two pin points at opposite corners. Then replace the pins, and pull the cords until the two parts of the six-inch cord lie upon two sides of the parallelogram. When this is the case, the third cord will be found to be equal in length, and lie in the same line as a diagonal of the parallelogram.

A number of methods can be devised for illustrating the same principle of the parallelogram of forces illustrated by the preceding exercises. Two spring balances may be connected by a string and a mass hung from the string, or a three-way string may be arranged as in the following exercises.

86. Other Proofs of Parallelogram of Forces.

(*a*) Procure a board about one foot square, with a pulley turning freely upon a smooth nail at each top corner. Pin a sheet of paper upon the board. Tie a thin wire hook at each end of a piece of fine string about a yard long, and at the middle of the string tie another piece of string about a foot long, with a wire hook at the free end. A three-way string is thus obtained.

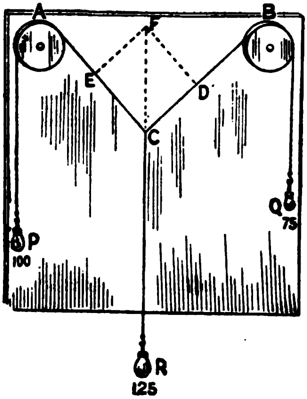

FIG. 119.—Board fitted with pulleys, for demonstrating the parallelogram of forces.

Round each of the two pulleys *A* and *B* pass one of the long strings. Upon one hook hang say 75 grams, upon the other 100 grams, and upon the hook connected to the middle of the string hang 125 grams. When the masses are at rest, draw lines upon the paper in the directions taken up by the strings, making them all meet at *C*. Now remove the strings, and measure off from the point *C* the distances *CD*, *CE* proportional to the masses *Q* and *P*. The distance *CD*, for instance, could be 75 mm. and the distance *CE* 100 mm.

Complete the parallelogram by drawing *DF* parallel to *CE* and *EF* parallel to *CD*. Draw the diagonal *CF*, and measure it. The length will be found proportional to the mass *R*.

(*b*) Repeat the experiment with various masses in bags hanging from the three hooks, and observe that in each case (*a*) the direction of the diagonal *CF* is vertical, (*b*) the number of units of length in the diagonal bears the same proportion to the sides of the parallelogram that the mass *R* bears to the masses *P* and *Q*. Express your results as follows:

$\frac{P}{R}$.	$\frac{CD}{CF}$.	$\frac{P}{Q}$.	$\frac{CE}{CF}$.

The results will show clearly that so far as strength or *magnitude* is concerned the three forces due to the masses *P*, *Q*, and *R* are represented by the sides and diagonal of the parallelogram *CDFE*. As to *direction*, the lines *CD* and *CE* were drawn in the directions in which the forces act with reference to the point *C*. These forces are kept in equilibrium by the force due to the mass *R* acting vertically downwards. The resultant of the two forces must therefore be proportional to *R* and act vertically upwards, and this is what you have found.

You have now proved by several experiments the principle of the parallelogram of forces, viz.: *If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of the two forces will be represented in magnitude and direction by that diagonal of the parallelogram which passes through this point.*

This principle is so important that it should be thoroughly understood. The following exercise will serve to test your knowledge of it:

(*c*) Hang a mass of 100 or 200 grams from a string. Pull the string out of the vertical by means of a thread attached to it and to a spring balance, the direction of the thread being horizontal (Fig. 120). Notice the reading of the balance. The mass *W* is kept from falling by two forces *P* and *Q*. You know the value of *P* and also of *W*. Find, by constructing a parallelogram and measuring its sides and vertical diagonal, the value of Q, that is the tension of the string.

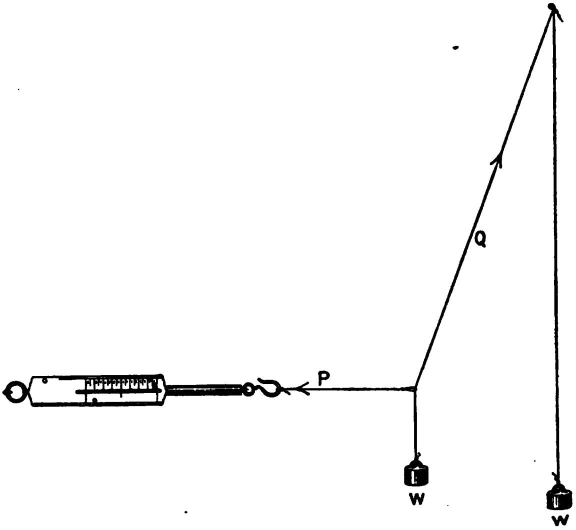

FIG. 120.—Three forces in equilibrium.

Repeat the experiment with the string pulled out of the vertical more than before.

87. Principle of the Triangle of Forces.

(*a*) Arrange the elastic cords upon a flat surface as previously done and stretch them (Fig. 121). Mark the position of C with a pin point. Remove the pins and draw the lines AC, BC, DC. Mark off AE, BF, DG equal to 2 inches. Produce EC to H, making CH equal in length to EC. Join GH.

The line GH will be found equal to CF. The three sides of the triangle CGH taken in order, as shown by the arrows, thus represent in magnitude and direction—***but not in position***—the forces acting upon the particle C.

Repeat the experiment with the cords stretched differently.

The principle thus illustrated can also be demonstrated by connecting two spring balances with a string and hanging a mass from the string, or by means of a three-way string arranged as in the experiment on the parallelogram of forces.

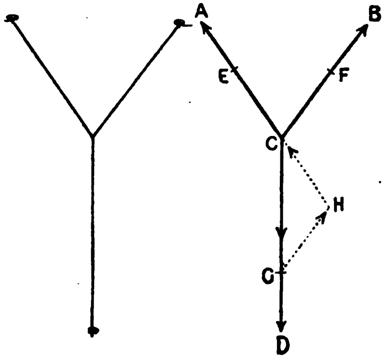

FIG. 121.—Experiment to illustrate the principle of the triangle of forces.

(*b*) Arrange three masses in equilibrium from the hooks on the three-way string as before. When the masses are at rest, draw lines from the point *C* in the directions of the three forces in equilibrium (Fig. 122). Now take the strings off the pulleys, and

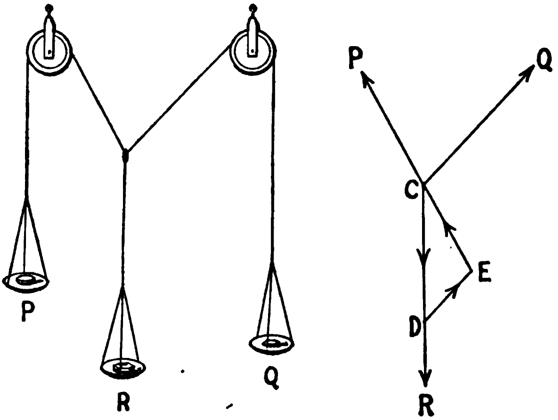

FIG. 122.—Experiment to show that three forces in equilibrium can be represented by the sides of a triangle.

measure from C a distance CD to represent the weight of the mass R. From the point D draw DE parallel to CQ, and produce PC to E.

The three sides of the triangle CDE thus represent the directions, but not the positions, of the three forces acting at the point C, and in equilibrium. Measure the sides CE and DE; their lengths will be found proportional to the weights of the masses P and Q.

Repeat the experiment with other masses in equilibrium.

The experiments show that when three forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle taken in order. The converse of this is true, as will be seen by the following experiment:

(*c*) Draw any triangle ABC. Measure the lengths of the sides, and put masses in the trays or bags on the three-way string in the same proportion as these lengths, the greatest amount being put in the bag connected with the middle string.

Arrange these strings on the boards with pulleys, as before; they will be found to be in equilibrium.

In the triangle ABC produce AC to D, making CD equal to AC. From C draw CE parallel and equal to AB. Place this figure behind the strings on the pulley-board, with the longest line BC vertical, and the point C at the point where the three strings meet.

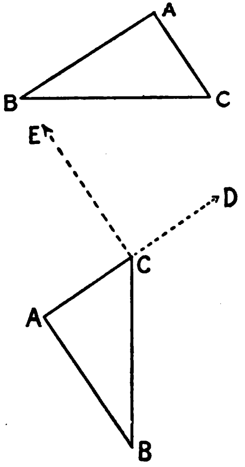

FIG. 123.—Experiment to show that the sides of a triangle can be represented by forces.

The directions CD, CE will be found to coincide with the directions of the strings connected with the masses P and Q.

You have therefore proved the triangle of forces, viz.: *When three forces acting at a point can be represented in magnitude and* *direction by the sides of a triangle taken in order, they are in equilibrium.*

88. Friction.

(*a*) Fix a small staple, such as is used to fasten wire netting, into one end of a rectangular block of wood. Place the block upon a smooth table, and connect an india-rubber cord or spring dynamometer with it by means of a bent pin. Hold the dynamometer horizontal, and slowly pull it until the block moves. Notice the amount by which the india-rubber or the spring is stretched when this happens. Repeat the experiment several times upon the same part of the table, and notice that the force required to move the block is about the same in each case.

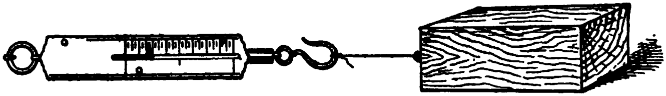

FIG. 124.—Measurement of friction.

Repeat the preceding exercise with the block upon a slate or a sheet of plate glass instead of upon the table. Excellent results can be obtained with roughened glass and blocks coated with cartridge paper.

> The surfaces upon which the block was placed in the two preceding exercises were comparatively smooth, so the *friction* between the block and the surfaces was not very great. Try the experiment with the block upon a sheet of sand-paper.

(*b*) Exercise 88 (*a*) proves to you that the amount of friction varies with different surfaces in contact with one another. To test whether the *size* of the surface of contact has any effect upon the amount, repeat the experiments with the block upon one of its side faces, taking care that the direction of the grain of the wood is the same as before. It will be found that the same pull or force is required to move the block when the surface in contact is small as when it is large.

(*c*) Another method of studying the effects of friction is by inclining the board or table upon which the friction block rests. Obtain a smooth board about 18 in. long and 4 in. wide, and, if possible, connect one end of the board to another similar strip of wood by means of a hinge. The upper smooth board can be inclined to the other by pushing a block between the two. Place the rectangular block already used upon the board, and gradually raise the unhinged end of the board until the block begins to slide. Measure the perpendicular height to which the board can be raised before motion commences. Repeat the experiment with the block on the side face used before.

(*d*) Fix a mass upon the block, and again find the amount of tilt necessary to make the block begin to slide.

> You have increased the pressure upon the board, and, if the friction remain the same, you would expect that the block would not slide so easily. But if the friction increases in just the same proportion as the pressure increases, evidently the weighted block will slide just as easily as the block by itself. Do your results bear out this relation between pressure and friction?

Repeat the experiment, using a slate instead of the board.

89. Coefficient of Friction.

(*a*) To determine the exact relation between the amount of friction and the pressure between the surfaces in contact, cut several pieces of sheet lead of sizes such that with the block they have masses respectively of 100, 200, 300, and 400 grams. Place the block upon the board with the lead, which with it has a mass of 100 grams, upon it. Connect an india-rubber cord, or better, a spring dynamometer, with the block, and determine as before the pull required to overcome the friction.

Repeat the experiment with the other pieces of lead upon the block, one after the other, making several observations of the pull required in each case.

> If you use an india-rubber cord for the experiments, it will be necessary to find what weights will stretch it to the

extent observed in each case. This can be done by holding the cord vertically, and gradually putting weights in a light pan hung from the hook at the free end. It is better, however, to use a dynamometer having graduations to show the force indicated by a certain extension. In either case you will find what forces expressed as weights are required to move the block when the surfaces in contact are under different pressures. Tabulate your results as below:

PRESSURE = MASS OF BLOCK + LEAD.	LIMIT OF FRICTION OR PULL REQUIRED TO MAKE THE BLOCK MOVE.	$\frac{\text{FRICTION}}{\text{PRESSURE}}$.

The ratio of friction to pressure, that is $\frac{\text{friction}}{\text{pressure}}$, is known as the *coefficient of friction*, and is roughly constant for any two given surfaces.

90. Principle of Inclined Plane.

Force acting Parallel to the Plane.

(*a*) Obtain a large pill-box or small canister with a cover and run a blanket pin or knitting needle through it from end to end, to form an axle (Fig. 125). Fit a wide wire handle upon the axle, and fasten to it the india-rubber cord or dynamometer previously used. Place some shot or sand in the roller thus constructed. Pull the roller along the board used in the preceding exercise, with the board horizontal, and notice how much the cord is stretched. Now tilt the board slightly and pull the roller up it, as in Fig. 126, taking care that the direction in which you pull is parallel to the slope of the board. Observe the amount

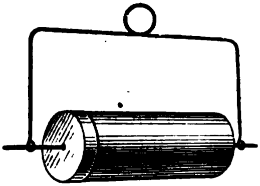

FIG. 125.—Roller for inclined plane, made from a tin canister.

by which the cord stretches as you pull up the roller. Then hold the cord vertically and find what load will stretch it by the same amount. The weight of this load represents the force used. Weigh the roller and so obtain the mass moved. Afterwards measure the length *AC* and the height *AB*, and find the relation between them as below:

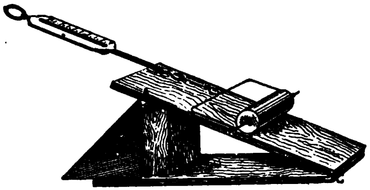

FIG. 126.—Inclined plane. Effort exerted parallel to plane.

> It does not matter what length of the plane is measured, so long as the perpendicular line to the base from the upper point of the length used is taken as the height. A convenient way to make the measures is to place a set square on the base as shown in Fig. 126, so that the top is just on a level with the plane on which the roller moves. The length of the plane is then taken as the length *AC*, and the height is taken as *AB*.

$$\frac{\text{Mass moved}}{\text{Force exerted}} = \frac{\ldots\ldots}{\ldots\ldots} =$$

$$\frac{\text{Length of plane}}{\text{Height of plane}} = \frac{\ldots\ldots}{\ldots\ldots} =$$

Are the two results approximately the same? If so, you may conclude that the mechanical advantage of an *inclined plane*, when the force exerted acts parallel to the plane, is equal to the length of the plane divided by the height.

(*b*) Repeat the experiment with the inclined plane, that is the board, tilted at different angles, and with different amounts of shot in the roller. Record in columns as below:

LOAD.	FORCE.	HEIGHT OF PLANE.	LENGTH OF PLANE.	$\frac{\text{LOAD}}{\text{FORCE}}$.	$\frac{\text{LENGTH}}{\text{HEIGHT}}$

Force acting Horizontally.

(*c*) Your experiments have shown you that, in the case of the inclined plane, when the power acts parallel to the plane

$$\text{Mechanical advantage} = \frac{\text{length of plane}}{\text{height}}.$$

Fix the board at any convenient height and calculate the mechanical advantage by finding the ratio of the height to the length of the plane. Afterwards test whether the calculated result agrees with the result obtained by practically finding the ratio of the force exerted to the mass moved in the case of the inclination chosen.

(*d*) Arrange the roller as shown in Fig. 127. Keeping the india-rubber cord or spring balance horizontal, pull the roller up the inclined plane. Observe the stretch of the india-rubber, and find what load will produce the same amount; or, if you are using a dynamometer, observe the reading under these conditions. Then weigh the roller with the wire handle. Measure the lengths of *BC* and *AC*.

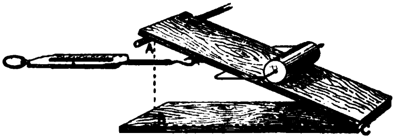

FIG. 127.—Inclined plane. Effort exerted horizontally

$$\text{Mechanical advantage} = \frac{\text{load moved}}{\text{force exerted}} = \frac{\ldots\ldots}{\ldots\ldots} =$$

$$\frac{\text{Base of plane}}{\text{Height of plane}} = \frac{\ldots\ldots}{\ldots\ldots} =$$

Compare the results thus obtained.

(*e*) Repeat the preceding experiment with the board tilted at different angles. Record your results as below:

Load Moved.	Force Exerted.	Height of Plane.	Base of Plane.	$\frac{\text{Load}}{\text{Force}}$.	$\frac{\text{Base}}{\text{Height}}$.

This experiment and the preceding one will enable you to understand the action of a *Wedge*, which may be considered as two inclined planes base to base. In this case the plane is pushed forward by a power acting parallel to the common base. The principle is therefore just the same as when an object is pulled up an inclined plane by a power acting parallel to the base, but the direction of action is reversed.

More exact experiments illustrating the principle of the inclined plane may be performed with the following apparatus :

91. Exact Measures with Inclined Plane.

Force acting Parallel to the Plane.

(*a*) From a thick glass tube cut off two pieces each a foot or so in length, and two pieces about an inch long. From thick brass wire which will go into the tubing cut off two pieces about six inches long. Put one of the short glass tubes at the middle of each wire and then bend the wire twice at right angles. Now put the ends of the wires into the long glass tubes, so as to form the oblong figure about two inches wide, shown in Fig. 128. Fasten one of the short tubes with the wire in it to a strip of wood about 15 inches long, as in Fig. 129.

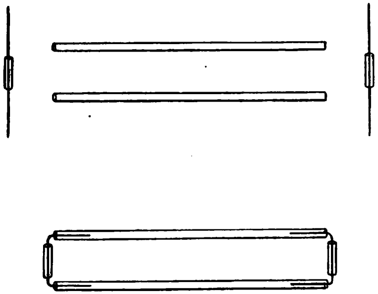

FIG. 128.—Stages in the construction of an arrangement for inclined plane experiments.

The long glass tubes can be given any inclination by moving the support shown along the base.

Fit corks into the ends of a piece of glass tubing, about three inches long, and pass a knitting needle or a piece of one through them to form an axis. Bend a piece of wire to form a handle for the roller thus produced, making the distance between the two sides of the wire greater than the distance

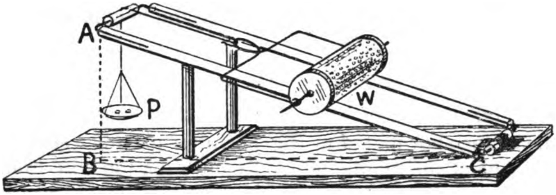

Fig. 129.—Glass inclined plane. Effort exerted parallel to plane.

from the outer edge of one long glass tube to the outer edge of the other (Fig. 130).

Now set the glass tubes to any convenient inclination. Take out one of the corks, and after putting some shot in the tube replace the cork. Place the roller *W* upon the inclined tubes, and add a sufficient load to the tray *P* (Fig. 129) to keep it from running down. Weigh the roller *W* and its contents, and *P* and its contents. Also measure the length *AC* of the plane of the tubes, and the height *AB*.

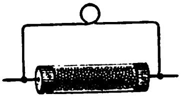

Fig. 130.—Glass roller for inclined plane.

After making these measures, tabulate your results thus :

W.	*P.*	Length, *AC.*	Height, *AB.*	$\frac{W}{P}$.	$\frac{AC}{AB}$.

(*b*) Make several experiments with the tubes at different inclinations and different loads in the roller, and then calculate the ratio of *W* to *P* and of *AC* to *AB*, as indicated in columns 5 and 6.

The ratio will be found approximately to agree with one another, thus showing that when an object is kept in position upon an inclined plane by a force exerted parallel to the direction of the plane, the mass of the object bears the same proportion to the force used to keep it in position that the length of the plane bears to the height.

Force acting Horizontally.

(*c*) Cut off a piece of wire 9 or 10 inches long, and bend it into the shape shown at *A* in Fig. 131, after placing a short piece of glass tube at the middle of it. Remove the top wire from the tubes already used, and put the newly bent wire in its place as indicated. Set the tubes at any convenient inclination, place the roller, with shot in it, upon them, and keep it in position by putting a load in the tray *P*. With this arrangement the force which keeps the roller *W* from running down the plane acts horizontally, that is, parallel to the base of the plane instead of parallel to the length. Determine the values of *W* and *P* for several inclinations, taking care that in each case the force due to *P* acts horizontally upon *W*. Measure the lengths *BC* and *AB* for each experiment, and tabulate your observations as follows:

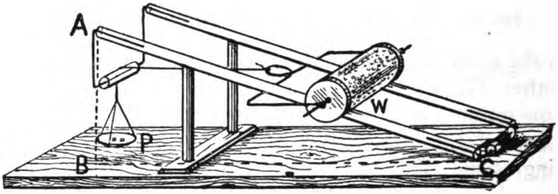

FIG. 131.—Glass inclined plane and roller. Effort exerted horizontally.

W.	*P*.	BASE, *BC*.	HEIGHT, *AB*.	$\frac{W}{P}$.	$\frac{BC}{AB}$.

A comparison of columns 5 and 6 will show you that when the force keeping an object on an inclined plane acts horizontally, the ratio which it bears to the mass of the object is the same as the ratio of the height of the plane to the base.

92. Pressure upon an Inclined Plane.

(*a*) To determine the pressure exerted by an object upon an inclined plane when in equilibrium, incline the glass tubes to a convenient angle, say 45°, and place a thin knitting needle across them. Hang a tray *W* from the needle, as shown in Fig. 132; place some shot or lead in it, and keep the needle from rolling down by means of shot in *P*. When the needle is in equilibrium attach an india-rubber or spring dynamometer of the needle, and stretch it slowly, with the cord or spring perpendicular to the plane, until you can just lift the needle off the plane. Notice the reading of the dynamometer when you do this. Make several observations of the force required, and take the mean.

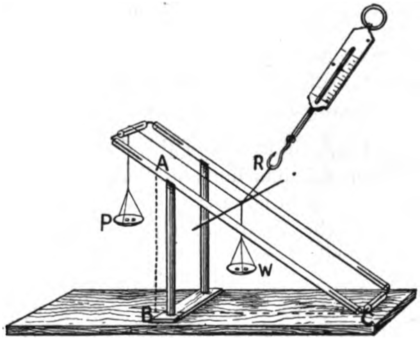

FIG. 132.—Experiment to determine the pressure upon an inclined plane.

You see from this experiment that the needle is kept in equilibrium by three forces *P*, *W*, and *R*. Before the dynamometer was connected with the needle the force *R* was the pressure on the plane, and you measure this pressure by means of the dynamometer.

(*b*) Represent these conditions graphically, as shown in

Fig. 133. Draw the angle *ACB* equal to the angle of the plane. Draw *OP* parallel to *AC*, and of a length equal to the number of units in the force *P*. Make *OR* perpendicular to *OP*, and of a length equal to *R* units; draw *OW* perpendicular to *BC*, and with a length equal to *W* units.

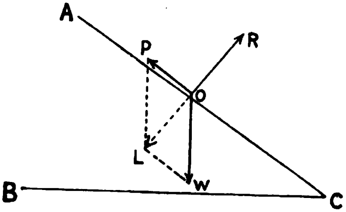

FIG. 133.—Graphic representation of conditions of equilibrium upon an inclined plane.

Construct the parallelogram *OPLW* by drawing *PL*, *WL* parallel to *OW*, *OP* respectively. Draw the diagonal *OL*, and measure its length. Compare the length of the resultant *OL* thus found graphically with the value of *R* determined by the dynamometer.

(*c*) Find, by means of the dynamometer as before, the value of *R* when *P* acts horizontally. Represent your results graphically, as in Fig. 134, and compare the length of *OL*, determined by measurement, with *OR* given by the dynamometer.

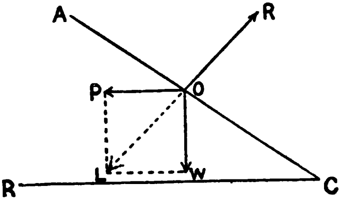

FIG. 134.—Graphic representation of conditions of equilibrium upon an inclined plane.

(*d*) In Exercise 86 (*b*) on the parallelogram of forces you have found the strength and inclinations of certain forces in equilibrium. Select one of the sets of measures, and arrange the inclined plane so that *P* is equal to the smaller of the component forces, and *W* is equal to the other component. Then try whether the force *R* indicated by the dynamometer is equal to the resultant of the parallelogram of forces previously determined.

CHAPTER XI.

MOTION, AND THE FORCE OF GRAVITY.

93. Production of Motion by Force.

(*a*) AT the top of an upright stand 7 or 8 ft. high fix a pulley (an aluminium pulley is convenient for the purpose) which is capable of moving with great ease, and spins for a considerable time before coming to rest.[1] Procure two cylinders, of iron, brass, or lead, having a hook fitted to one end of each, and a mass of about 100 grams each. Also obtain two similar cylinders of about 200 grams each. Suspend the lighter pair by a long thread over the pulley. The length of the thread should be such that when one cylinder rests on the ground the other is near the pulley. Next, by means of small additional pieces of lead, or other material, adjust the mass of one cylinder, *A*, so that, though it does not fall of itself, yet if started it slowly moves down with, as nearly as can be seen, uniform velocity.

Draw down the other cylinder, *B*, and on *A* place a piece of wire, hair-pin shape, having a mass from 2 to 4 grams. Release *B* and observe that *A* falls with a continually increasing velocity. Do the same experiment again, but pin to the upright the small platform *F*, which has a hole through it, through which the cylinder can pass but not the wire rider.

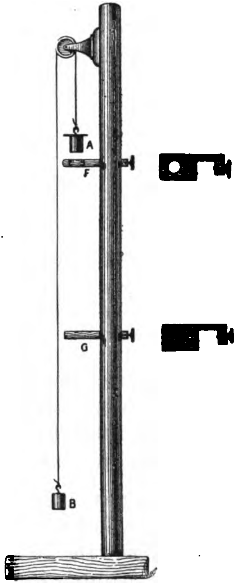

Fig. 135.—Apparatus for experiments on motion.

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[1] If an Atwood's machine is available, it can be used instead of the pulley and upright here described, but it is not essential.

By repeated experiments adjust the height of this platform so that you hear the rider strike it exactly 1 sec. after B has been released. When this is done, next adjust another platform, G, at such a height that the cylinder A strikes this platform exactly 1 sec. later.

Measure the distance from F to G.

Now move down the platform G so that 2 seconds elapse between the removal of the rider and the striking of the platform G, and again measure the distance FG. If possible also adjust for a time of 3 seconds.

Compare these spaces and the times thus :

SPACE $= s$.	TIME $= t$.	$\frac{\text{SPACE}}{\text{TIME}} = \frac{s}{t}$.

Hence deduce that the *space is proportional to the time*, that is, the *velocity is uniform*. This may be represented by the expression $s = vt$, where s represents the space, v the velocity, and t the time.

(*b*) Remembering that velocity is the space passed in one second, what velocity was produced in the preceding experiment, by what force was it produced, and for how long did this force act on the system?

(*c*) Repeat the preceding experiment, using now the larger pair of cylinders.

Compare the velocities given in 1 second by the added rider. Are they equal or approximately so?

Multiply the total mass moved in each experiment by the velocity given in 1 second. Compare the products so obtained and observe they are approximately equal. This product is called *momentum*.

94. Relation between Momentum and Force.

(*a*) Find, as in the preceding exercises, the momentum produced by a rider of known mass in 1 second with (1) the smaller cylinders, (2) the larger cylinders. Compare the two values. Does the momentum produced in 1 second by a constant force depend on the mass moved? If your work has been carefully done you will find it does not.

(*b*) Repeat the experiment, using riders of different masses, and record the momentum produced in 1 second by each rider. Calculate your results thus, and compare them:

WEIGHT OF RIDER, *i.e.* FORCE CAUSING MOTION.	MOMENTUM PRODUCED IN 1 SECOND.	$\frac{\text{MOMENTUM}}{\text{FORCE}}$.

(*c*) Repeat the experiments with different cylinders and see the same relation holds good.

You will find that the momentum produced in 1 sec. is proportional to the force which causes the motion, and may be taken as a measure of this force.

The British unit of force is the poundal, and the metric unit is the dyne; they are defined as follows:

The poundal is a force which, acting on a mass of one pound for one second, gives it a velocity of one foot per second.

The dyne is a force which, acting on a mass of one gram for one second, gives it a velocity of one centimetre per second.

95. Variation of Velocity with Time.

(*a*) Arrange the apparatus as in Exercise 93 (*a*), so that the rider is removed by *F* after a fall of 1 sec., and the cylinder strikes *G* 1 sec. later. Measure *FG*.

Now adjust *F* so that the rider is not removed until 2 seconds after *A* has started, the time of fall from *F* to *G* being still 1 second; and then alter so that a fall of 3 seconds is attained. Measure *FG* in each case. This distance is evidently a measure of the velocity produced in respectively 1, 2, and 3 seconds. Compare the results:

TIME OF FALL.	VELOCITY PRODUCED.	$\frac{\text{VELOCITY}}{\text{TIME OF FALL}}$.
1 sec.		
2 sec.		
3 sec.		

Observe that the velocity produced is proportional to the time, that is $v=at$, where a is the velocity given in 1 second.

(*b*) Repeat the experiments, using different riders.

(*c*) Repeat, using different cylinders.
Satisfy yourself that the above deduction is still correct.

The velocity given per 1 second is called the *acceleration*. In the above experiments the acceleration is *uniform*, that is, the same velocity is added in each second.

What was the *momentum* produced per second in each case? Observe that since the *acceleration* is *uniform* it is the same in each second, so that the *force* is *constant*; and hence you will see that *uniform acceleration* is characteristic of a *constant force*.

We may therefore say, force = mass × acceleration, this product being identical with the momentum produced per second.

(*d*) What was the acceleration in (*b*) and in (*c*)? What would have been the velocity produced in 10 seconds in each case?

96. Space in Relation to Time.

(*a*) Arrange the apparatus as in Exercise 93 (*a*), but only one platform is needed. Adjust its height so that the cylinder falls for 1 sec., and measure the space fallen in this time. Then adjust for a fall of 2 secs., and again measure; also for 3 secs.

Compare these spaces and the times thus:

SPACE.	TIME.	$\frac{\text{SPACE}}{\text{TIME}}$.	(TIME)^2.	$\frac{\text{SPACE}}{(\text{TIME})^2}$.
	1		1	
	2		4	
	3		9	

It will be found that the space traversed is proportional to the square of the time.

(*b*) Repeat Exercise (*a*), using a different pair of cylinders. Does the connection between space and time still hold good?

(*c*) Repeat Exercise (*a*), using different riders. Does the connection again hold good?

(*d*) Draw a curve to show the connection between space fallen and time taken in each of the Exercises (*a*), (*b*), (*c*).

(*e*) From your results calculate the distance the cylinder would traverse in 10 secs. Is it 10 times that passed in 1 sec? If not, how many times greater is it?

97. Space in Relation to Acceleration and Time.

(*a*) Arrange the apparatus as in Exercise 93 (*a*) and measure the distance fallen 1 sec. before and after the removal of the rider, that is, the distances *AF* and *FG*. The first is the measure of the *mean* velocity, the latter that of the final velocity. Compare these two velocities.

(*b*) Repeat the experiment, using different riders and also the different cylinders. Tabulate your results:

FINAL VELOCITY.	MEAN VELOCITY.	$\frac{\text{FINAL VELOCITY}}{\text{MEAN VELOCITY}}$.

Observe that final velocity = 2 × mean velocity.
Therefore mean velocity = $\frac{1}{2}$ final velocity.
As, however, space = mean velocity × time,
hence space = $\frac{1}{2}$ final velocity × time.
But final velocity = acceleration × time;
therefore $s = \frac{1}{2}at^2$.

98. The Weight of a Gram in Dynamical Units.

(*a*) Place a small rider on the cylinder *A*, using the 100 gm. cylinder, so that *A* falls. Measure the time, t_1, taken for a fall of 200 cms.

Add another rider of known mass, and again find the time, t_2, for the same fall.

Then calculate the acceleration by the method already described.

$$s = \tfrac{1}{2}a_1t_1^2;$$

therefore $$a_1 = \frac{2s}{t_1^2} \text{ and } a_2 = \frac{2s}{t_2^2}.$$

The total mass was, say *m* gms. in the first case, and the velocity produced per second was a_1. Therefore the momentum produced per second was $m \times a_1$. The additional load *w* increased the acceleration to a_2; therefore momentum produced per second in the second case was

$$(m+w)a_2,$$

and the momentum produced per second *by the load of w gm.* was

$$(m+w)a_2 - ma_1 = m(a_2 - a) + wa_2.$$

Therefore 1 gm. produced per second $= \frac{m(a_2 - a) + wa_2}{w}$ units of momentum and is equal to this number of *dynes*.

(*b*) Repeat the preceding exercise several times with different riders and cylinders, and take the mean of your results as the measure in dynamical units—dynes—of the weight of 1 gram.

The value so obtained is probably not very accurate, but should be within 10 per cent. It may be determined more accurately by another method. If the weight of the mass w had only acted on the mass itself, that is, if the mass had been falling freely, the velocity would have been greater. Suppose 1 gram falls freely, and in 1 sec. acquires a velocity of g ft. per sec., then the momentum produced is $1 \times g$ units of momentum, and 1 gram $= g$ dynes.

99. Determinations of the Intensity of Gravity.

(*a*) The value of g, or the velocity produced per second by a body falling freely under the action of the force of gravity, can be determined very satisfactorily by the following ingenious device.[1]

An upright frame (Fig. 136) about 3 feet high has a ledge screwed at the back so that it can be supported at the edge of a bench. Near the top of the frame a small platform of hard wood, having a slot cut in it, is screwed. Upon this platform rests the knife-edge of a pendulum formed by a bar of wood about 4 feet long and 1 inch square section. The pendulum is cut away at the top to enable it to swing freely in the slot of the platform. Holes at right angles to the plane of oscillation are bored in the pendulum for the insertion of lead plugs to alter the mass centre and so vary the time of vibration. A silk thread, to which a double conical bob is attached, is connected with the pendulum, and passes over two light pulleys, as shown in the illustration. Small boxwood pulleys, bushed with glass tubing, and pivoted on steel hat pins with large heads, work very well. The sharp equator of the bob is smeared with Brunswick black, or the lower part of the face of the pendulum may be given a thin coating of paraffin wax.

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[1] A rough form of apparatus for determining the value of g by the method here described was shown by Mr. S. Whalley at the conference of Science Teachers, held in London in January, 1899. To Mr. Whalley belongs the credit of experimentally developing the method, but he does not claim to have been the originator of it. The neat arrangement shown in Fig. 135 represents the form in which the apparatus is used in Nottingham Science Schools. The illustration and particulars referring to it have kindly been furnished by Mr. W. R. Swain, one of H.M. Inspectors of the Department of Science and Art.

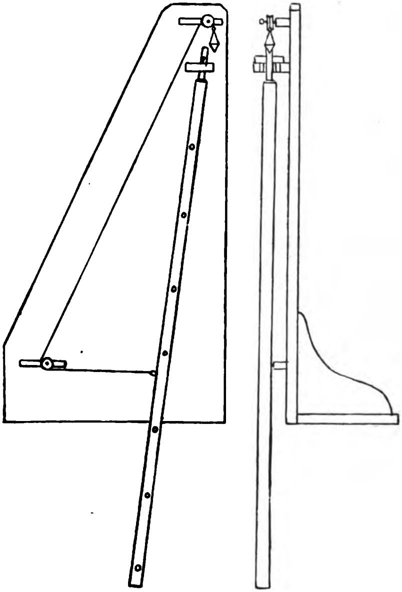

FIG. 136.—Apparatus for the determination of the acceleration due to gravity.

To perform an experiment with this apparatus, the pendulum is adjusted so that the face when vertical touches the equator of the bob. Pull the pendulum a little out of the vertical as shown in the diagram (Fig. 136). Measure the height in feet of the equator of the bob above any convenient base below the pendulum. Then burn the thread at a point just above the bottom pulley. The pendulum is thus released, and the instant it reaches the vertical line it is struck by the sharp equator of the falling bob, and a mark is thereby made upon it. Measure the distance in feet from the base to the top of this mark when the pendulum has come to rest. The difference gives the space passed through by the bob in one-quarter of a complete swing-swang. Determine the time taken by 10 or 20 to-and-fro vibrations of the rod swinging freely, and calculate from it the time (t) of $\frac{1}{4}$ of a vibration. The value of the force of gravity (g) can then be determined from the simple formula:

$$s=\tfrac{1}{2}gt^2,$$

so that

$$g=\frac{2s}{t^2}.$$

(*b*) Repeat the experiment by varying the rate of vibration of the pendulum and making the measurements in centimetres.

Another method of indirectly determining the value of g is by means of a pendulum.

(*c*) Suspend a metal ball about an inch in diameter from a thread or fine wire about a yard long. The upper end of the thread should be clamped between two square-edged clips or pieces of metal, so that the pendulum thus formed may have the same length in all parts of its swing.

Place a thin upright rod in front of the pendulum hanging vertically. Start the pendulum swinging through a comparatively small arc. Stand in front of the rod so that you can see easily when the pendulum passes. With a stop-watch in hand, notice the pendulum passing the rod, and count say 100 swings, or 50 vibrations if a complete to and fro movement is reckoned as one. When the pendulum passes the rod for the hundredth time, moving in the same direction as at the beginning of the observation, take the time again. The time of one complete vibration can be obtained by dividing the whole of the interval by half the number of swings past the rod.

It has already been shown, Exercise 80 (*b*), that the time of vibration of a pendulum varies as the square root of the length. The time of oscillation can, by a simple application of dynamical principles, be shown to be given by the expression:

$$\text{Time of oscillation}=2\times 3\tfrac{1}{7}\sqrt{\frac{\text{length}}{\text{gravity}}},\ \text{or}\ t=2\pi\sqrt{\frac{l}{g}};$$

where t stands for the time of vibration, l for the length of the cord, g for the value of the accleration due to gravitation, and π for the ratio between the circumference and diameter of a circle.

From this equation it is easy to obtain an expression for the value of g. Thus, squaring both sides, we have

$$t^2 = \frac{4\pi^2 l}{g},$$

so that

$$g = \frac{4\pi^2 l}{t^2}.$$

Using the observations you have made as to the time of a single swang of your pendulum, and taking as the length of the pendulum the distance in feet from the bottom of the clamp holding the thread to the centre of the ball, determine by means of this equation the value of g.

(*d*) Determine the value of g as in the preceding exercise, but make the measurement of length in centimetres.

100. Motion of Ascending and Falling Bodies.

(*a*) Drop a marble, piece of wood, and other things of about the same size from various heights, and adjust the height so that the object takes 1 sec. to reach the ground or table upon which it is dropped. The value found will probably be between from 14 to 18 ft., or from 9 to 10·5 metres. This is evidently not an accurate method of determining the space described in one second owing to difficulties of obtaining the time nearer than to about ·1 or ·2 secs., but the fall will be found about that stated, and the same value will be found for the different objects. If a high window is available, endeavour to find the height fallen in 2 secs., and see that it is about 4 times that fallen in 1 sec.

If a body is thrown into the air it rises to a certain height and then falls. Compare the time of ascent and descent as follows:

(*b*) Two observers are required. One should ascend to a window and drop a glass marble on to a stone floor. It will be heard to strike; and it rebounds, rises, and again falls. The time taken for this rise and fall is the interval between the two sounds of the marble striking the floor. This can be accurately determined by one observer. The second observer notes the height to which the marble rises. Calculate the time of fall for this height; subtract the result from the total interval observed, and so obtain the time of rise. Repeat the experiment several times, and tabulate your results:

Time.	Height.	Time of Fall.	Time of Ascent.
.... Secs.	.... Ft.	.... Secs.	.... Secs.

Compare the time of fall with time of ascent. What do you observe?

101. Motion down an Inclined Plane.

(*a*) Slightly tilt a long board by blocks placed under one end. Place on it a marble; observe that the marble rolls down the board with a continually increasing velocity. (If the marble does not run straight, a narrow **V**-shaped groove should be cut along the length of the board.)

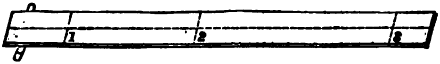

Fig. 137.—Grooved board for experiments on motion down an inclined plane.

(*b*) Adjust a block of wood on the board so that the marble strikes it 1 second after it is released, and measure the distance. Do the same for 2 seconds and for 3 seconds.

Observe that, as in the case of the falling cylinders, Exercise 96 (*a*), the space described is proportional to the square of the time.

Alter the height, and repeat the experiment.

Repeat, using different marbles or a cylinder of brass.

Calculate the *acceleration* in each case.

(*c*) Using different marbles, find the space they will roll through in 3 seconds, and observe that this is independent of the size of the marble.

(*d*) Find the time taken by the marbles to roll the length of the board. From this calculate the acceleration by the formula $s=\frac{1}{2}at^2$.

(*e*) Find by a number of experiments the distance, s, the marble rolls in 1 second, and measure in each case the height, h, to which you have raised one end of the board above the other. Calculate your results thus:

SPACE.	HEIGHT.	$\frac{\text{SPACE}}{\text{HEIGHT}}$.

Observe that the space passed in one second is proportional to the height. From this it follows that the acceleration is proportional to the height.

(*f*) Find the time required for the marble to roll down the board, measuring, as before, the height to which the upper end is raised.

Do this several times, varying the inclination of the board. Tabulate your results thus:

HEIGHT (h).	TIME (t).	PRODUCT ($h \times t$).	TIME^2.	PRODUCT ($h \times t^2$).

Observe that the product in column 5 is approximately constant, thus showing that the square of the time is inversely proportional to the height of fall.

Does this follow immediately from the result of experiment (*e*) by the application of the formula known?

CHAPTER XII.

THERMOMETERS, AND SIMPLE OBSERVATIONS WITH THEM.

102. Construction of a Thermometer.

(*a*) OBTAIN a piece of thermometer tubing, about ten inches long, and if you have practised glass-blowing, blow a bulb about ½ inch in diameter at one end. If you cannot do this, it will save time to obtain a tube with a bulb already blown upon it.

Mode of Filling the Bulb and Tube.

(*b*) Gently heat the bulb of the thermometer tube over a flame, and then quickly immerse the open end in alcohol coloured with cochineal.

> A small quantity of the spirit enters the bulb to take the place of the air driven out by the heat.

If your thermometer tube has not a cup blown at the open end, like that in Fig. 138, connect the open end with a glass funnel by means of a piece of india-rubber tubing about an inch long. Pour into the cup or funnel sufficient coloured alcohol to fill the bulb. Hold the bulb in a flame (Fig. 139) until nearly all the alcohol is boiled away, then take away the flame. Alcohol will enter the bulb and stem. Repeat the operation until the bulb and stem are filled with the liquid; then disconnect

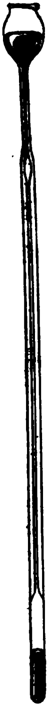

FIG. 138.—A thermometer in course of construction.

the funnel or pour the surplus alcohol out of the cup, and let the instrument cool.

Sealing the Tube.

(*c*) When the alcohol in the stem has contracted two or three inches, melt the tube near the end in a blow-pipe flame, and draw it out to a narrow neck about eight inches from the bulb. Immerse the bulb and stem in hot water—*not* boiling water—and when the alcohol fills the tube, seal the tube quickly by directing a small blow-pipe flame against the narrow neck, and afterwards complete the sealing so as to make the end strong. The final result should be like one of the instruments in Fig. 140.

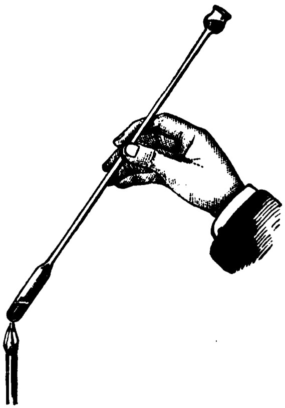

FIG. 139.—Method of filling a thermometer.

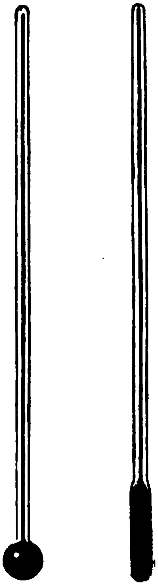

FIG. 140.—Thermometer tubes filled and sealed ready for graduation.

You have now an instrument by means of which the state of warmth or coldness of a body can be indicated. When it is warmed the liquid in it rises in the tube, and when it is cooled the liquid sinks, so that by noticing the position of the top of the liquid you are able to determine whether the thing in contact with the instrument is warm or cold. Marks or graduations can be made upon the tube to indicate various degrees of warmth, and when this has been done the instrument becomes a *thermometer*, that is, an instrument for the determination of temperature, or intensity of heat.

103. Simple Temperature Observations.

(*a*) Examine the thermometer supplied you (Fig. 141). Notice that it is similar to the instrument you have made, but is graduated or marked in a certain manner. The divisions are called *degrees*.

(*b*) Hold a thermometer in your mouth for a short time; find and record the degree of temperature indicated by it. Also place a thermometer under your arm-pit; find and record the temperature there.

(*c*) Let a thermometer hang freely in the air for a short time, and find the degree of temperature of the air indicated by it. Then place the thermometer upon your table, and see whether the same temperature is indicated.

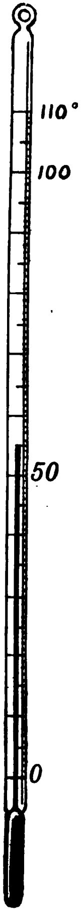

FIG. 141.—A thermometer with Centigrade graduations.

104. Freezing Point of Water.

(*a*) Support a funnel upon one of the rings of your retort-stand, and fill it with scraps or shavings of ice (Fig. 142). Place the thermometer in the funnel, pack shavings or scraps of ice around it, and notice how the mercury behaves. Observe the exact point at which the mercury ceases to contract. The experiment shows the temperature at which ice melts and water freezes. Repeat the experiment with fresh ice and see if the point at which the mercury stands is the same as before.

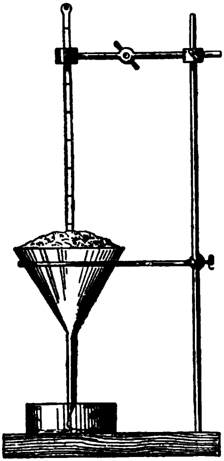

FIG. 142.—Determining the freezing point of a thermometer.

(*b*) Place some small pieces of ice in a beaker of cold water. Well stir the water with the thermometer, and observe its temperature. Gently heat the beaker of water until the ice is melted, stirring all the time. Describe fully the changes of temperature that occur both before and after the ice has melted. You will find that the temperature will not rise until all the ice has melted.

105. Boiling Point of Water.

(*a*) Fit a cork, with two holes through it, into a flask or test-tube (Fig. 143). Half fill the test-tube with water, and push the thermometer through one of the holes in the cork until the bulb is wholly immersed. Into the other hole, fit a piece of glass tubing bent at right-angles. Place the test-tube upon the retort-stand, and observe the division of the thermometer scale level with the top of the mercury. Gently heat the water until it boils, noticing how the mercury is affected throughout the process. See where the mercury stands when the water is boiling, and find whether any difference is produced when the water is boiling

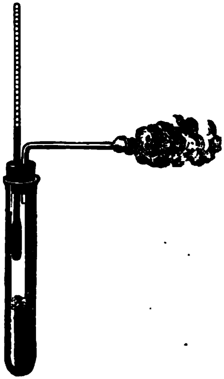

FIG. 143.—Determination of boiling point.

furiously, and when it is boiling gently. Raise the thermometer until the bulb is just above the boiling water, and observe the temperature which it then indicates.

The temperature of boiling water depends upon the pressure of the atmosphere, and therefore varies with the height of the barometer, and with the altitude of the water above sea-level. Under the same conditions, however, the temperature of pure boiling water is the same. The standard conditions are when the water is at sea-level and the barometer stands at a height of 30 inches. The mercury in an accurately graduated thermometer should, if held in boiling water under these conditions, be at the division 100, or 212, according to the system of graduation used. More exact methods of determining the boiling point of a liquid are described in later Exercises.

106. Degrees of Temperature.

The difference of temperature between the boiling point and freezing point of water could be divided into any number of steps, but only two methods of subdivision are in general use. These two thermometric scales are the Centigrade scale and the Fahrenheit scale. A third scale, the Réaumur scale, is used for domestic purposes in Germany, but is not employed in this country. The three scales are shown in Fig. 144.

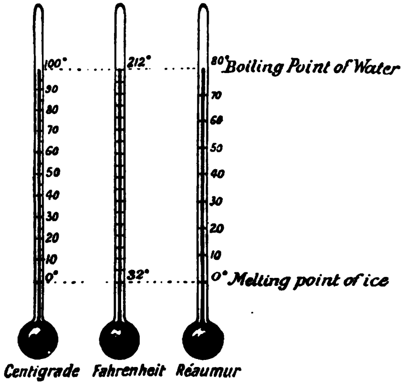

FIG. 144.—Thermometric scales

The Centigrade Scale.—Here the freezing point is called *zero*, or *no degrees Centigrade*, written 0° C. The boiling point is called *one hundred degrees Centigrade*, and is written 100° C. The space between these two limits is divided into 100 parts, and each division called a *degree Centigrade.*

The Fahrenheit Scale.—On thermometers marked in this way the freezing point is called *thirty-two degrees Fahrenheit*, written 32° F., and the boiling point *two hundred and twelve degrees Fahrenheit*, written 212° F. The space between the two limits is divided into 180 parts, and each division is called a *degree Fahrenheit.*

The Réaumur Scale.—The freezing point is 0° as on the Centigrade, and the boiling point is 80° R. The degrees are thus longer than on a Fahrenheit or a Centigrade thermometer of the same size.

107. Conversion of Thermometric Scales.

The interval between the boiling and freezing points, that is, the same temperature difference, is divided into 100 parts on the Centigrade scale and 180 parts on the Fahrenheit, and consequently 100 Centigrade degrees are equal to 180 Fahrenheit degrees, which is the same as saying one degree Centigrade is equal to nine-fifths of a Fahrenheit degree, or one degree Fahrenheit is equal to five-ninths of a degree Centigrade.

100° C. = 180° F.; ∴ 5° C. = 9° F.; ∴ 1° C. = $\frac{9}{5}$° F., and 1° F. = $\frac{5}{9}$° C.

In converting Fahrenheit readings into Centigrade degrees, we must subtract 32 (because of what has been said of the freezing point on the former scale) and multiply the number thus obtained by 5 and divide by 9. To change from Centigrade to Fahrenheit, multiply the former reading by 9 and divide by 5 and add 32 to the result.

EXAMPLE.—What temperature on the Fahrenheit scale corresponds to 20° C.?

Answer. 20° C. is 20 C. degs. above temperature of melting ice, *i.e.* 20 × $\frac{9}{5}$ Fahr. degs. above 32° F = (36 + 32)° F. = 68° F.

When it is necessary to refer to temperatures lower than the freezing point of water, a minus sign is placed before the temperature, thus three degrees below the freezing point of water on the Centigrade scale is written – 3° C.

(*a*) Using this information, make the necessary calculations and fill up the following spaces:

85° C. correspond to - - - ......° F.
50° C. „ „ - - - ......° F.
26° C. „ „ - - - ......° F.
10° C. „ „ - - - ......° F.
4° C. „ „ - - - ......° F.
– 5° C. „ „ - - - ......° F.

Also fill up the following blanks:

200° F. correspond to - - - ......° C.
180° F. „ „ - - - ......° C.
100° F. „ „ - - - ......° C.
75° F. „ „ - - - ......° C.
40° F. „ „ - - - ......° C.
10° F. „ „ - - - ......° C.

108. Graduation of an Alcohol Thermometer.

The alcohol thermometer you have constructed may be graduated by comparison with a centigrade thermometer. One way of doing this is to stick a strip of blank paper on the tube by means of gum, or you may cover the tube with paraffin wax. Place the thermometer in a funnel containing shavings of ice, and make a mark on the paper or wax level with the top of the alcohol in the tube. Then plunge both the centigrade thermometer and the one you wish to graduate into cold water in a beaker. Warm the water and watch the thermometer. Make marks on the paper where the coloured liquid stands as the thermometer reads 10°, 20°, 30°, . . . and number these marks in the same way on the paper or wax, 10°, 20°, 30°. . . . Transfer these marks to the tube itself with a three-cornered file, or fix the thermometer upon a strip of wood having paper glued upon one face, and make the degree marks upon the paper.

109. Graphic Representation of Thermometer Readings.

Read your thermometer at a fixed time every day, and plot the observations from day to day on a chart, as with the barometer readings, Exercise (56*a*). The degrees can be written at the ends of the horizontal lines of squared paper, and the days of the month can be written at the top or bottom of the vertical lines as in Fig. 145.

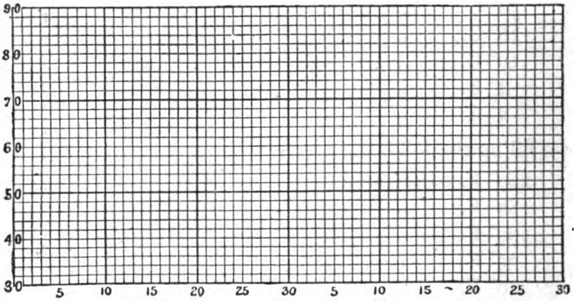

FIG. 145.—Form for recording daily temperature observations.

110. Effects of Salts in Altering the Freezing and Boiling Points of Water.

Freezing Point.

(*a*) Place a few shavings of ice or pieces about the size of a pea in a funnel, supported as in Fig. 142, or a beaker. Sprinkle the ice with salt; then add some more ice, and sprinkle again with salt. Repeat this operation several times, making the proportions of the two substances about 1 part of salt to 4 parts of ice, and then stand in the mixture a thermometer graduated below the freezing point of water. The temperature of the mixture will be found to be below 0° Centigrade. Record the lowest temperature reached.

(*b*) Put a thin test tube, containing a little water, into a mixture of ice and salt. Stand the thermometer in the water, and observe that the temperature never sinks below 0° Centigrade or 32° Fahrenheit while the water remains liquid. You should not, however, leave the thermometer in the water. If you continue the experiment long enough, the water will freeze, and the ice formed will cool down towards the temperature of the freezing mixture of ice and salt.

(*c*) Put some salt water into the test tube instead of fresh water, and stand the test-tube in a mixture of ice and salt. You will find that the water can be cooled below 0° C. without turning into ice.

(*d*) Find the temperature produced when crystallised calcium chloride is mixed with ice.

(*e*) Mix two parts by weight of ammonium nitrate with one part of ammonium chloride, and dissolve the mixture in cold water. Observe the temperature before and after putting the salts in the water.

> In every case where salts are present in solution, the freezing point of water is lowered. Pure ice ought, therefore, to be used in determining the freezing point of a thermometer.

Boiling Point.

(*f*) Add a little salt to water. Put the salt water in a test-tube or flask, fitted as in Fig. 143, and, using a thermometer graduated above the boiling point of water, find the temperature, (i.) of the steam from the boiling salt solution, (ii.) of the boiling liquid itself. Record your results. Compare with Exercise 102 (*a*). Why was the thermometer held in the steam? Repeat the exercise, using calcium chloride instead of salt.

111. Determination of Melting Point.

(*a*) Draw out, in a blow-pipe flame, a piece of glass tubing so as to make a small thin-walled tube, about 2 or 3 inches long and about ·1 inch in diameter. In this put some finely powdered sulphur and fix the tube to the bulb of the thermometer (it will probably stick to it if moistened; if not, it may be secured with a platinum wire or an india-rubber ring). (Fig. 146.) Place the thermometer bulb with the lower part of the tube in a beaker of sulphuric acid or glycerine with the sealed end of the tube and the bottom of the thermometer at the same distance above the bottom of the beaker. Heat *gently* with a small flame. The sulphuric acid is kept at a uniform temperature by raising and lowering the curved stirring rod shown in the figure. Watch the sulphur carefully, and directly you see it to be melting note the temperature recorded by the thermometer, that is, the melting point of the sulphur.

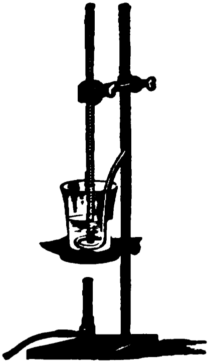

FIG. 146.—Determination of melting point.

Melting point of sulphur, - ......° C.

(*b*) Similarly find the melting point of bee's-wax. Cut off a length of the small thin-walled tube and dip it into some melted wax. In this way the fine tube becomes filled with wax which soon solidifies. Proceed as before.

Melting point of bee's-wax, - ......° C.

(*c*) Melt a little paraffin wax in a beaker, and immerse the bulb of a thermometer in the liquid. When the thermometer is taken out, a thin film of liquid paraffin will be seen upon it. Let the bulb cool, and notice the temperature when the wax assumes a frosted appearance, indicating that it is solidifying. When the wax on the bulb has became solid place the thermometer in a beaker of water and gently heat the water. Observe the temperature at which the wax becomes transparent again. The mean of this result and the preceding one is the melting point of paraffin wax.

112. Determination of Boiling Points.

(*a*) Put a little methylated spirit in a test-tube, and gradually heat it in a beaker of water until it boils. Find the temperature of the boiling spirit and of the vapour, and record the results.

(*b*) A convenient arrangement for determining the boiling point of a liquid is shown in Fig. 147. The vapour passes up into the bulbs and out of the side tube, its temperature being shown by a thermometer, the bulb of which is adjusted just below this outlet. If available, use this apparatus to determine the boiling points of water, turpentine, milk, beer, vinegar, and whisky, or of as many of these liquids as you can obtain.

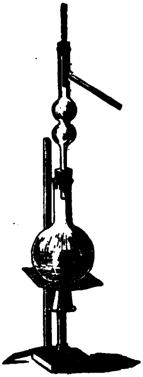

FIG. 147.—Flask fitted with tube for the determination of boiling point.

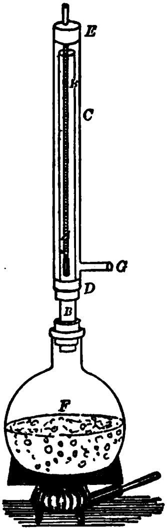

FIG. 148.—Apparatus for the accurate determination of boiling points.

On account of the condensation of vapour upon the thermometer, the method used in the preceding exercises to determine boiling points is not a very accurate one. More exact determinations of the boiling point can be made by means of the apparatus shown in Fig. 148. A can or flask *F* is fitted with a cork, through which a glass or brass tube *B* passes. Surrounding this tube is a wider tube *C*, fitted upon the inner tube by means of a piece of thick india-rubber tubing *D*. At the top of the outer tube is a cork *E* having a hole in which a thermometer can be fitted. When the water in the flask is boiled, steam passes up the inner tube *B*, and down the wide tube *C*, and escapes at the outlet *G* into the open air.

(*c*) To use the apparatus, gently push the top of the stem of the thermometer into the cork which fits in the outer tube, adjusting it so that the 100° point is just below the cork. Fit the cork in its place, boil the water, and when steam has been coming off for about a quarter of an hour, raise the cork and read the thermometer. Repeat the observation after a few minutes, and when two readings obtained at an interval of about ten minutes agree, record the observation. The temperature you observe is the boiling point of water under the particular conditions existing at the time and place of the experiment.

113. Anomalous Expansion of Water.

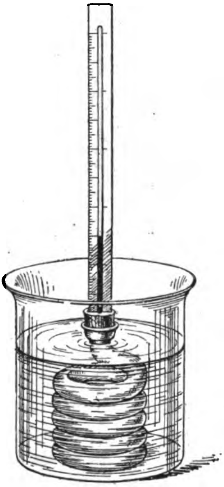

FIG. 149.—Apparatus for the determination of the changes of volume of water near the freezing point.

(*a*) Make a coil of lead or "compo" tubing. Fit an india-rubber stopper, or a good cork, having a glass tube of narrow bore through it, into one end of the coil. Suck boiled water into the coil until it can be seen in the glass tube above the stopper, and then fit a good stopper into the other end of the coil (Fig. 149). Fasten a scale to the glass tube, and then place the coil with the tube and scale into a beaker of water at the temperature of the room, and hang a thermometer in the water.

Notice the position of the surface of the water in the tube, and the temperature of the water in the beaker. Add shavings of ice, or pounded ice, to the water, and when the temperature is steady again notice the position of the top of the water. Continue the cooling with ice, making observations of the position of the surface for about every three degrees down to 1° C. Then let the water in the beaker gradually rise in temperature, adding a little warm water, if necessary, and again observe the positions at the same temperatures as before. The mean of the two positions observed for each temperature should be taken as the true reading for that particular temperature. Construct a curve like that shown in Fig. 150 to represent your observations of the changes of volume of water at temperatures near the freezing point.

At what temperature has the water in the coil the least volume, and therefore the maximum density?

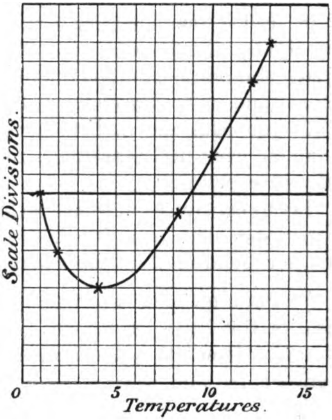

FIG. 150.—Graphic representation of changes in volume of water near the freezing point.

The changes in volume of water, when cooled down to freezing point, can also be observed by means of a test-tube of water having a good stopper, with a narrow glass tube in it instead of the coil used in the preceding exercise. But as the expansion of water between 4° C. and 0° C. is very small, the experiment has to be carefully performed to be successful.

114. Density of Water and Ice.

(*a*) Half fill a burette or graduated glass tube with paraffin oil. Put the burette in a jar containing ice and water until the temperature reaches 0° C. Observe the level of the paraffin oil in the burette. Gently drop in pieces of dry ice, and observe the volume of the ice added by noticing the rise of level of the oil. Let the ice melt, and when it is converted into water, again observe the reading of the burette. This reading will give you the volume of water produced by the melting of a certain known volume of ice. Find the ratio of the volume of the water produced to that of the ice from which it came. Hence the result will give you the density of ice.

(*b*) Fill a small narrow-necked bottle with water, and cork it up tightly. (A bulb blown at the end of a glass tube is better.) Place the bottle in a bowl and cover it with a freezing mixture of ice and salt. Carefully cover the bowl with a duster or cloth until you hear the bulb burst, then take off the covering and examine what has happened. Describe and explain the result of your experiment.

Try to explain, from what you have learnt by your experiments, why water pipes burst in winter. How is it that the burst pipes are not found until the thaw sets in?

(*c*) Put a few pieces of ice in a glass of water. Does the ice float or sink? Is ice heavier or lighter than an equal bulk of water? How do you account for the difference of density between ice and water?

(*d*) Cut a piece of wax candle into chips, and put them into a test tube. Melt the wax by gently heating the test tube, and while it is liquid throw in two or three small pieces. Do these pieces float or sink? Can you tell from your observations whether wax contracts or expands when passing from the liquid to the solid state?

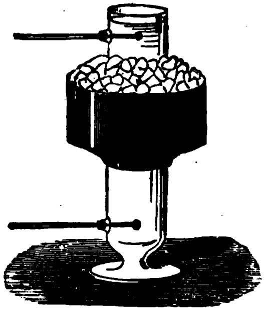

FIG. 151.—Hope's apparatus for the determination of the temperature of maximum density of water.

115. Maximum Density of Water.

When water contracts its density must increase, and the degree at which contraction ceases and expansion commences marks therefore the point of *maximum density of water.* The changes of density near the freezing point can be indirectly observed by means of Hope's apparatus (Fig. 151), the principle of which is based upon the fact that the denser parts of a liquid sink and the lighter rise to the top. The apparatus consists of a glass or metal vessel having a trough surrounding the middle part. Two thermometers are inserted in holes near the top and bottom of the instrument respectively.

(*a*) To use the apparatus, fill the inner cylinder with cold water—at a temperature of 6° C. to 8° C.—and the trough with a freezing mixture. Notice the readings of the thermometer at intervals of a minute or two, and record as below.

TIME.	TOP THERMOMETER.	BOTTOM THERMOMETER.

At first the bottom thermometer will be most affected, while the top one remains at nearly the same degree. When the temperature of 4° C. is reached, the bottom thermometer ceases to fall, but the upper one commences to fall rapidly, and continues to go down until a temperature of 0° C. is reached if the experiment is continued long enough.

Explain the changes observed by means of the facts you have learnt, as to the variations of the density of water with temperature.

The apparatus for performing this experiment can easily be made in a workshop. Holes are cut in the side of a tin canister, and short tin tubes are soldered into them. Corks for holding the thermometers are fitted into the tubes. For the trough a large tin canister is obtained and reduced to a convenient height. A hole is then cut in the bottom to permit the trough to fit upon the outside of the smaller canister. This must, of course, be done before the top tube is soldered on. To obtain the best results with this or any other form of Hope's apparatus, the cylinder should be wrapped in cotton wool and stand on similar material; and the observer should only stand near it when he wishes to read the thermometer.

CHAPTER XIII.

CONDUCTION AND RADIATION OF HEAT.

116. Relative Conductivities of Metals.

(*a*) Twist an iron and a copper wire, about 10 cm. long, at one end, and put the twisted end in a small flame. After a short time the wires become too hot to hold. Why? Which wire reaches this stage first?

(*b*) Obtain wires of copper, iron, brass, German silver, and of any other metals available. Let the diameters be the same as nearly as possible, and the lengths about 15-20 cms. Fasten the wires upon a strip of wood as shown in Fig. 152). Support the wood in a horizontal position and heat the wires with a flame where they meet. After a few minutes slowly move a safety match along each wire in succession, commencing at the ends away from the flame, and notice the points at which the matches will light. Repeat the experiment several times; then take away the flame and measure the distance of these points from the heated ends. Find the average distance for each wire.

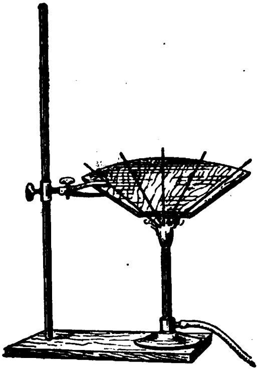

FIG. 152.—Experiment to show difference of conductivity of heat along metal rods.

The numbers obtained will show roughly the relative conducting powers of the metals of which the wires are composed. Write down the names of these metals in the order of their ability to conduct heat, beginning with the best conductor, and putting against each name the average distance at which the match was ignited upon it.

(*c*) With the small cylinders of metal provided and your steel scale (if not too heavy) perform the following experiment. Fix the scale upon one of the cylinders of metal by means of a small lump of bee's-wax. Place the cylinder on a warmed plate of iron, supported on a tripod as shown in Fig. 153. Heat is conducted by the cylinder to the wax, which eventually melts, causing the scale to fall over. Repeat the experiment with the different cylinders and note the number of seconds taken for the wax to melt.

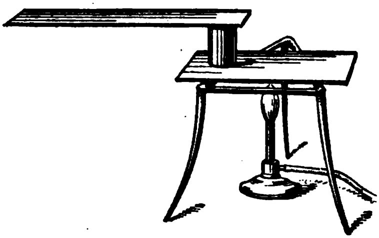

FIG. 153.—A conductivity experiment.

	No. of seconds in which wax melts.
(*a*) Copper, - - - - -	......
(*b*) Brass, - - - - -	......
(*c*) Bismuth, - - - - -	......
(*d*) Hard wood, - - - -	......

As some metals take more heat than others to raise them to the same temperature, the results obtained will not accurately show the difference of conducting power of the cylinders used.

(*d*) Bore a number of holes in a cork bung of such a size that several rods of different material but the same length and thickness just fit. Push these rods through the cork so that they all protude equally. Cover the longer parts with wax by means of a brush or cloth dipped in melted wax. Now put the cork bung on to the top of a tin canister or beaker in which water is being boiled in such a way that the shorter unwaxed parts of the rods dip into the water. Notice that the wax melts further along some rods than others. Allow the rods to remain until no further melting is observed, and then measure each rod from the extremity to the beginning of the unmelted wax. The conductivities of the metals are proportional to the squares of the distances along which the wax is melted.

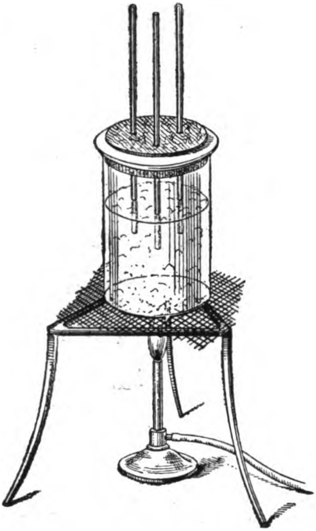

FIG. 154.—Rods heated by hot water to show difference of conductivity.

The following effective arrangement for determining the relative thermal conductivities of metals has been devised by Mr. Edwin Edser.[1]

(*e*) Procure a piece of brass tube, about 10 cms. in diameter and 20 cms. in length. Close one end by means of a brass disc. Bore a number of holes in this disc to receive the extremities of rods of copper, brass, iron, etc., each rod being 2·5 mm. in diameter and about 15 to 20 cms. in length. Solder the rods in position perpendicular to the disc.

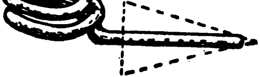

FIG. 155.—Enlarged view of index of Edser's conductivity apparatus.

Upon each rod place a small index, made from a piece of copper wire of about ·8 mm. diameter, bent into the form shown in Fig. 155, a small arrow-head of blackened paper or mica being attached by shellac varnish. The rings forming part of each index should be wound on a rod *very slightly* larger in diameter than the experimental rods.

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[1] *Nature*, July 13, 1899.

To perform an experiment, invert the brass vessel; slip an index on each rod, the single ring (Fig. 155) being left in contact with the disc, and melt a very small amount of paraffin wax round the rings. Support the vessel with the rods downwards, as in Fig. 156. The solid wax will hold the indexes in position. Now pour boiling water into the brass vessel.

When that part of a metal rod, in the neighbourhood of the double ring of the index, reaches the melting temperature of the wax, the index commences to slip downwards, carrying the wax with it, and when the temperatures of the rods have acquired steady values, the indexes will have descended to points on the various rods where the wax just solidifies, and which, therefore, possess equal temperatures. The conductivities of the rods are proportional to the squares of the distances from the bottom of the brass vessel to the respective positions indicated by the several arrow-heads.

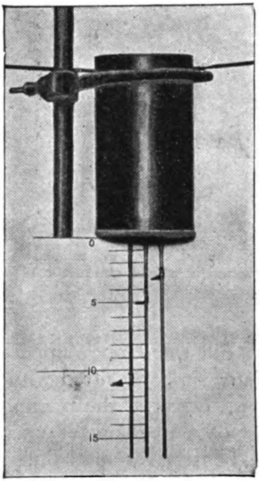

FIG. 156.—Edser's apparatus for demonstrating the relative thermal conductivities of metal. (The left-hand rod is of copper, the middle one of brass, and the right-hand one of soft steel.)

Observe the positions of the indexes on the rods you use and determine the relative conductivities of the substances of which they are composed.

117. Relative Conductivities of Liquids.

(*a*) Fill a narrow test-tube three quarters full with cold water, and having weighted a small piece of ice by winding wire round it, or in some other way, drop it into the test-tube. Hold the test-tube near the bottom where the piece of ice is, and warm the top of the water in a Bunsen flame, as shown in Fig. 157. The water at the top can be heated until it boils vigorously and yet the ice is not melted, showing what a bad conductor the water is.

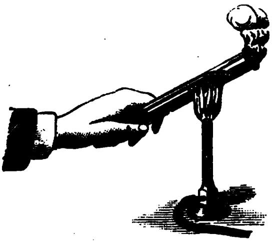

FIG. 157.—Experiment to show that water is a bad conductor of heat.

(*b*) Nearly fill a narrow test-tube with mercury and heat it at the top, as in the preceding exercise. You will find that the test-tube will soon become too hot to hold.

(*c*) Fill the Hope's apparatus, used in Exercise 115, or a tin canister similarly fitted with tubes for thermometers, with cold water. Observe the readings of the thermometers. Float the cover of a tin canister upon the top of the water, and pour a little benzene or methylated spirit into it. Ignite the spirit, and notice that the readings of the thermometers remain practically unaltered, and that even the top one is not affected until the spirit has been burning for some time.

118. Radiation and Absorption.

(*a*) Obtain two small bright tin cans or canisters, and fit into each a cork having a hole through which a thermometer will pass. Cover the outside of one of the vessels with lampblack by holding it over a candle or luminous gas flame, or over burning camphor. Put the same quantity of hot water at the same temperature in each, and then cork up the vessels, each cork having a thermometer through it so that the bulb is well immersed in the water. Observe the temperature of each vessel of water, and if one is higher than the other, cool the vessel until the temperatures are equal. Then put the vessels in a cool place where there are no draughts; and after 20-30 minutes again read the temperatures.

The blackened vessel will be found to have lost or radiated more heat than the bright one.

(*b*) Similarly equally fill a blackened and a bright vessel with cold water of the same temperature, and hang them for 20-30 minutes at the same distance above an iron plate, supported on a tripod stand and heated by a laboratory burner. At the end of this time observe their temperatures.

The blackened vessel will be found at a higher temperature than the bright one, therefore a lamp-black surface absorbs heat better than a bright metallic surface.

119. Construction of a Differential Air-Thermometer.

(*a*) Fasten two flasks or bulbs together with air-tight joints, by a tube bent six times at right angles. Before fixing this tube in position put in some coloured liquid, so that it fills the bend as in Fig. 158.

Notice that whenever one bulb is at a higher temperature than the other the level of the liquid in the parallel tubes is altered, the liquid approaching the bulb with the lower temperature.

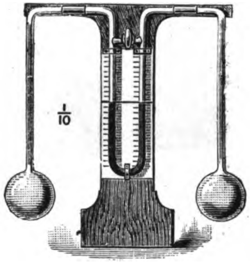

FIG. 158.—A differential thermometer.

120. Emissive and Absorptive Powers of Different Surfaces.

(*a*) Cover one bulb of a differential thermometer with lamp-black, and the other with tinfoil. Place the thermometer so that the bulbs are at equal distances from a vessel containing hot water. Notice and explain the movement of the liquid in the thermometer.

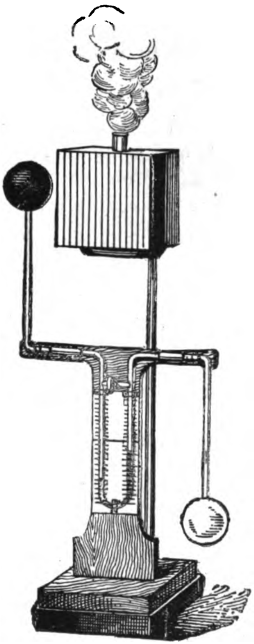

FIG. 159.—Differential thermometer arranged for radiation experiments.

(*b*) Blacken one of the bulbs of a differential thermometer. Turn up this bulb into the position shown in Fig. 159. Obtain a small square biscuit box or vaseline tin. Cover one of the vertical faces with lamp-black, upon another paste a piece of white paper, and roughen another by rubbing it with coarse emery cloth. Fill the box with hot water, and support it on a retort stand near the upper bulb of the differential thermometer, beginning with the bright unaltered side facing the bulb. Observe the depression of the liquid in one of the parallel tubes of the thermometer. Turn all the faces successively towards the upper bulb, taking care that the distance is the same in each case, and observe the effect produced.

Which of the faces radiates most heat? Write down the names of the surfaces in the order of their radiating power, as indicated by your observations.

121. Rate of Cooling.

(*a*) Obtain a small tin canister, place it in a large beaker of water, and put enough shot in it to sink it until the top is a little higher than the top of the beaker, when the water-level outside is about that indicated in Fig. 160. Observe and record the temperature of the water. Bore a hole in a slab of wood and insert a cork, having a hole through which a thermometer will pass.

Hold the thermometer in the hot air rising from a flame until the temperature shown by it is 70° to 80° C. Then, and not before, push the thermometer through the cork—the top of the stem first—and place the wood on the top of the beaker, so that the bottom part of the thermometer is enclosed in the tin vessel. Observe the temperature every half-minute as the thermometer cools, until its temperature has fallen within about 10° of that of the water. Record the observations as indicated below.

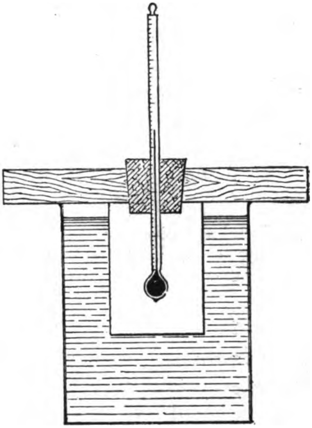

FIG. 160.—Experiment on the rate of cooling.

TEMPERATURE OF THERMOMETER.	EXCESS OVER TEMPERATURE OF WATER.	AVERAGE EXCESS.	FALL IN DEGREES PER HALF-MINUTE.	$\frac{\text{AVERAGE EXCESS}}{\text{FALL}}$

The average excess is obtained by dividing the sum of two successive numbers in the second column by 2; it is the mean of the excess at the beginning and end of the half-minute.

Newton's law of cooling states that *if an object is enclosed in a chamber of uniform temperature the rate at which it loses heat is proportional to the excess of the temperature of its surface above that of the chamber.* The numbers in the last column ought therefore to be the same if the law were strictly true, but as it only holds good when the excess is small, you will find that the numbers will differ slightly.

ADDITIONAL EXERCISES.

(L.U., London University; L.C.C., London County Council Intermediate Scholarship Examination; J.O., Junior Oxford; J.C., Junior Cambridge.)

Length.

1. Measure the diameter and thickness of the metal disc provided.

2. Measure the diameter of the given piece of wire by two different methods.

3. Measure, by means of a piece of thread, the circumference of the lead pencil provided.

4. Determine the internal diameter of a glass tube by means of a cardboard wedge.

5. Find the length of the outline drawn upon the sheet of paper supplied.

6. Measure the distance between successive threads of the screw provided.

Area.

1. Draw, on the cardboard supplied, a square 10 cm. in the side, and in this square inscribe a circle. Then, by cutting out the cardboard appropriately and making use of the balance, determine the area of the circle in square centimetres.

(L.C.C. 1898.)

2. Find, by measurement, the area of the piece of tin plate; then with the help of the balance find its thickness, assuming its specific gravity to be 7·6. (L.C.C. 1897.)

3. Find the superficial area of a cricket ball.

4. Draw a square and measure its area. Construct, as nearly as you can, a circle having the same area as the square. Determine the length of the circumference of the circle and the total length of the sides of the square.

5. Construct an ellipse and prove by experiment that its area is 3⅐ times the area of the rectangle contained by its semi-axes.

Volume.

1. Find the volume of the given cylinder by measurement and also by a weighing method. (B.Sc. Lond. Pass. 1895.)

2. Determine, by direct experiments, both the radius and volume of the given sphere, and assuming the volume of a sphere of radius r to be $\frac{4}{3}\pi r^2$, use your result to find a value for π. (J. C. Local. 1897.)

3. Determine the average volume of the bullets contained in the packet *R*. (J. C. Local. 1898.)

4. Examine and describe the body *A*. Find its mass, its volume, and its density. (J. O. Local. 1897.)

5. Blow a bulb on a glass tube of small bore, and measure the capacity of the same and that of each cm. length of the stem. (Int. Sci. Hons. L.U. 1897.)

6. You are given a rough block of hard granite; explain carefully how you could determine accurately the volume of the block in cubic inches.

7. Find the volume of a cricket ball.

Density and Relative Density.

1. Dissolve 10 grams of common salt in 50 grams of distilled water. Then, with the help of the apparatus given, determine the density of your solution, and calculate its total volume. (L.C.C. 1898.)

2. Find by comparison with water, with the help of the apparatus given you, the specific gravity of the given liquid. (L.C.C. 1897.)

3. Prepare four determinate strengths of sugar solution in water and find their specific gravities by weighing a piece of metal in them. (Int. Sci. Hons. L.U. 1896.)

4. Find the specific gravity of some lead shot by a specific gravity bottle. (Int. Sci. Hons. L.U. 1895.)

5. Find the density of the given liquid. (Int. Sci. Hons. L.U. 1897.)

6. Verify at three or four points of its scale the graduations of a given hydrometer, by specific gravity bottle and balance, using in various dilutions the heavy aqueous solution provided. (Int. Sci. Hons. L.U. 1897.)

7. Examine and describe the body *A*. Find its mass, its volume, and its density. (J. O. Local. 1897.)

8. Determine the average density of the bullets in the packet *R*. (J. C. Local. 1898.)

9. Raise some water to the temperature 25°C., and dissolve in it such a quantity of the salt *A*, that a litre of the solution would contain 250 grams of the salt. Then determine the specific gravity of the solution at 25°C. (J. O. Local. 1898.)

10. Find the density of a cricket ball by first measuring its volume and mass, and then making a suitable calculation.

11. Determine the area and density of the rectangular piece of foil provided. Use your results to find the thickness of the foil.

12. Determine, by measuring the length and volume, the area of the cross-section of the piece of fine copper wire provided.

13. You are provided with a coil of wire, which must not be unwound. Measure the thickness of the wire, and determine the volume. Use your observations to find the length of the wire.

14. Find, by weighing and measurement, the diameter of the copper wire of specific gravity 8·9. (L.C.C. 1897.)

15. Determine the density of the piece of paraffin wax provided, and the mass of the small particles imbedded in another piece of wax of the same kind.

16. Determine the thickness of the hollow copper sphere provided, given that the density of copper is 8·9.

17. Find, by measuring and weighing in air, the density of the rectangular block supplied.

18. Determine the density of a cork by any method you consider the most suitable.

19. A wooden block having some nails driven into it, is supplied to you, and also a piece of the same wood and a few of the same kind of nails. Find the mass of the nails in the wood.

Boyle's Law.

1. Investigate the relation between p and v with air at constant temperature.

2. Verify in a Boyle's tube the relation between volume and pressure (at constant temperature) of the specimen of gas given you in a closed tube. (Int. Sci. Hons. L. U. 1897.)

Plotting of Curves.

1. Find the times of vibration of the given rod, used as a pendulum, for different points of suspension along the rod. Represent your results by a curve. (Int. Sci. Hons. L.U. 1895.)

2. Investigate the stretching of the given india-rubber cord, measuring both extension and diameter for different loads. Represent your results on section paper.

3. Plot on section paper the values found for l and t^2 with a simple pendulum of various lengths.

4. Draw a rectangle; construct a curve to show the relation between its length and breadth.

5. Plot a curve to show the relation between the radii and areas of circles having radii from 1 to 10 cm.

Mechanics.

1. You are provided with a box of weights, a lever, and an unknown mass. Determine by the principle of moments the unknown mass.

2. A balance having arms of unequal length is provided, and a box of weights. Without adjusting the balance, find the mass of the object supplied.

3. Find the mass of the lever supplied, without using a balance.

4. Find the centre of gravity of the piece of bent wire provided. Show, by means of a diagram, where the centre of gravity is situated.

5. Determine the centre of gravity of the irregular figure provided.

6. Construct a square. Divide it into four equal squares and cut away one of them. Find the centre of gravity of the remaining part.

7. Use the given pendulum to find the value of *g*.

8. Plot on section paper the values found for l and t^2 with a simple pendulum of various lengths. Deduce from your results the value of *g*. (B. Sc. L.U. Pass. 1895.)

9. The distances between the marks on the line drawn on the card *s* represent the spaces described in equal intervals of time by a body moving with uniform acceleration. Determine, from measurements, the acceleration and the final velocity of the body. (J. O. Local. 1898.)

10. Mark on the given lamina the position of its centre of gravity. (J. C. Local. 1897.)

11. Determine by direct experiment the resultant of two forces of 10 oz. and 15 oz. acting at a point in directions at right angles to each other. [The observations are to be recorded graphically]. (J. C. Local. 1897.)

12. Determine the law of friction in the given apparatus. (Int. Sci. Hons. L.U. 1897.)

13. Determine the amount of friction, for a series of loads, of wood on wood. (Int. Sci. Hons. L.U. 1896.)

14. Find the coefficient of friction $\left(\frac{\text{friction}}{\text{pressure}}\right)$ for the block provided and a slate.

15. Determine the limiting angle at which the block provided will rest upon an inclined strip of wood.

Heat.

1. Mix equal quantities of ice and salt, and ascertain the lowest temperature reached. Use the mixture to freeze water. (L.C.C. 1898.)

2. Determine the boiling point of the given liquid. (L.C.C. 1898.)

3. Determine the boiling point of water and deduce the pressure of the atmosphere, being given

Boiling point of water = 99° when barometer reads 733 mm.
" " = 100° " " 760 "
" " = 101° " " 787 "
(L.C.C. 1897.)

4. Determine the melting point of the given substance.
(L.C.C. 1897.)

5. Pour some hot water into the given vessel, and note the temperature every minute as it cools, keeping the water well stirred. Draw the curve of cooling on a sheet of squared paper, and compare the rates of cooling at the temperatures 65°, 55°, 45°. (L.C.C. 1897.)

6. Arrange an experiment to prove that a blackened metallic surface is a better radiator than a bright metallic surface.
(L.C.C. 1898.)

7. Determine the relative thermal conductivities of the three wires supplied.

8. Find the change of volume which a piece of paraffin wax experiences in melting.

INDEX.

[*The numbers refer to pages.*]

Abscissa, 104.
Absorption, 188; different powers of, 189.
Angles, measurement of, 134.
Archimedes, principle of, 77; direct proof of principle, 79; relative densities by principle of, 80.
Area, measurement of, 26-49; representation of, 27; comparison of British and metric, 28-30; determination of by squared paper, 30-33; of irregular figures, 35; determination of, geometrically, 33-37; of circle, 36; determination of, by weighing, 45-49.
Atmosphere, pressure of, 93.
Average, 7.

Balance, use of, 40-44; bent-lever, 117.
Barometer, siphon, 94; cistern, 96; reading and plotting heights of, 98; variation of, with altitude, 99.
Boiling point, determination of, 178.
Boyle's law, 100; graphic representation of, 109.
Burette, use of, 64.

Calipers, 14; slide, 20.
Capacity, measures of, 58-60; vessels used in measurement of, 61; relation between British and metric measures of, 63.
Centimetre, 3.
Centre of gravity, determination of, 122; uniform rod, 122; circular disc, 122; parallelogram, 123; triangle, 123; quadrilateral, 124; irregular figure, 124; open framework, 124; in relation to equilibrium, 125.
Circle, area of, 36, 45; by weighing, 46-47; circumference of, 9.
Conduction of heat, 184.
Conductivity, of metals, 184; Edser's experiment on, 187; of liquids, 188.
Cone, 56.
Conversion of units, length, 6, 7.
Cooling, rate of, 190; law of, 191.
Co-ordinates, 104.
Cubic measure, 51.
Curved lines, measurement of, 10-13.
Curves, plotting of, 104.
Cylinder, surface of, 39, 40; volume of, 54.

Decimetre, 3.

[The numbers refer to pages.]

Decimal system, 2.
Density, of solids, 68; of liquids, 69; bottle, 71; relative, by balancing columns of liquids, 72; of water and ice, 181; maximum, of water, 182.
Diameter, of sphere, 9; ratio to circumference, 8-10.
Displacement, volumes by, 61; in metric measure, 65.
Dynamometer, 138.

Expansion, anomalous, of water, 180.
Extension and tension, 136.

Flotation, 83.
Foot-rule, use of, 1-3.
Friction, 147; co-efficient of, 148.

Gram, definition of, 66; weight of, 162.
Gravity, centre of, 122; determination of, 163-166; specific, 72.

Hare's apparatus, 75.
Hydrometer, 85; of variable immersion, 86; simple, 86; Nicholson's, 86.

Length, measurement of, 1-25; metric measures of, 3, 4.
Lever, 110; inclined forces acting on, 112.
Loci, meaning of, 105; symbolic representation of, 107.

Mass, 41; standard, 42; units of, metric, 66.
Melting point, determination of, 177; of sulphur, 178; bee's-wax, 178; paraffin, 178.
Metre, 3.
Micrometer, screw, 21; gauge, 23.
Millimetre, 3.
Moments, principle of, 111.
Momentum, and force, 159.
Motion, production of, by force, 157; of ascending and falling bodies, 166; down inclined planes, 167.

Opisometer, 12.
Ordinate, 104.

Parallel forces, resultant of, 117; in equilibrium, 118; conditions of equilibrium, 119; three, 120.
Parallelogram, area of, 33; by weighing, 48.
Parallelogram of forces, 139-142.
Parallelopiped, volume of, 53.
Pendulum, 130.
Plane, inclined, 149-156; mechanical advantage of, 151; exact measures with, 152; pressure upon, 155; motion down, 167.
Pressure, of liquids, 88; relation to depth, 88; and area, 90; independent of form of vessel, 91; upward, 92; of vapours on barometric column, 97; and volume, relation between, in gases, 100.
Pulley, simple, 126; fixed, 126; movable, 127; tension of string in movable, 127; mechanical advantage of movable, 129; principle of work applied to, 130.

[*The numbers refer to pages.*]

Pyramid, 56.

Radiation, 188.
Resultant, of parallel forces, 117; definition of, 118.

Salts, effects of, on freezing point, 176; on boiling point, 177.
Scales, fractional, 13; diagonal, 15-17; conversion of thermometric, 174.
Screw, thread, 22.
Siphon, 76; barometer, 94.
Space, in relation to time, 161; in relation to acceleration, 161.
Sphere, surface of, 37-39; volume of, 57.
Spherometer, 24.
Steelyard, 115; Danish, 116.
Square, 26.

Temperature, degrees of, 173.
Tension and extension, 136.
Thermometer, construction of, 169; simple observations with, 171; graduation of, 175; graphic representation of readings of, 176; construction of differential, 189.
Tracing wheel, 12.
Triangle, area of, 34; by weighing, 48; of forces, 144-146; prism, 54.

Velocity, variation of, with time, 159.
Vernier, construction, 17; general rule for, 19; use of, 19.
Vibration of a lath, 132.
Volume, units of, 50-52; relation between British and metric, 52; determination of, by measurement, 52-55; comparison of, by weighing, 55; determination of, by geometry, 56-58.

Water, freezing point of, 171; boiling point of, 172.
Work, principle of, applied to levers, 114.

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